

Complex Number

0.1.2

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Contents

1	Namespace Index	1
1.1	Namespace List	1
2	Class Index	3
2.1	Class List	3
3	File Index	5
3.1	File List	5
4	Namespace Documentation	7
4.1	complex Namespace Reference	7
4.1.1	Detailed Description	7
4.2	math Namespace Reference	7
4.2.1	Detailed Description	7
5	Class Documentation	9
5.1	ComplexNumber Class Reference	9
5.1.1	Detailed Description	17
5.1.2	Constructor & Destructor Documentation	17
5.1.2.1	ComplexNumber	17
5.1.2.2	ComplexNumber	18
5.1.2.3	ComplexNumber	18
5.1.2.4	~ComplexNumber	18
5.1.3	Member Function Documentation	18

5.1.3.1	getImaginary	19
5.1.3.2	getReal	19
5.1.3.3	i	19
5.1.3.4	operator ComplexNumber< U >	19
5.1.3.5	operator*= operator*=	19
5.1.3.6	operator*= operator*=	20
5.1.3.7	operator+	20
5.1.3.8	operator+=	21
5.1.3.9	operator+=	21
5.1.3.10	operator-	22
5.1.3.11	operator-=	22
5.1.3.12	operator-=	23
5.1.3.13	operator/=	23
5.1.3.14	operator/=	24
5.1.3.15	operator=	24
5.1.3.16	operator=	25
5.1.3.17	polar	25
5.1.3.18	setImaginary	26
5.1.3.19	setReal	26
5.1.4	Friends And Related Function Documentation	26
5.1.4.1	abs	26
5.1.4.2	abs2	27
5.1.4.3	acos	27
5.1.4.4	acos	28
5.1.4.5	acosh	30
5.1.4.6	acosh	31
5.1.4.7	acot	32
5.1.4.8	acot	33
5.1.4.9	acoth	33
5.1.4.10	acoth	34
5.1.4.11	acsc	34

5.1.4.12	acsc	35
5.1.4.13	acsch	36
5.1.4.14	acsch	37
5.1.4.15	arg	38
5.1.4.16	asec	38
5.1.4.17	asec	39
5.1.4.18	asech	40
5.1.4.19	asech	41
5.1.4.20	asin	41
5.1.4.21	asin	42
5.1.4.22	asinh	44
5.1.4.23	asinh	45
5.1.4.24	atan	46
5.1.4.25	atan	47
5.1.4.26	atanh	48
5.1.4.27	atanh	49
5.1.4.28	cbrt	50
5.1.4.29	conjugate	51
5.1.4.30	cos	51
5.1.4.31	cosh	52
5.1.4.32	cot	54
5.1.4.33	coth	55
5.1.4.34	csc	55
5.1.4.35	csch	56
5.1.4.36	exp	56
5.1.4.37	inverse	57
5.1.4.38	log	57
5.1.4.39	log	58
5.1.4.40	log10	58
5.1.4.41	log10	59
5.1.4.42	logabs	60

5.1.4.43	logb	60
5.1.4.44	logb	61
5.1.4.45	logb	62
5.1.4.46	logb	63
5.1.4.47	nthRoot	64
5.1.4.48	nthRoots	65
5.1.4.49	nthRoots	66
5.1.4.50	operator!=	67
5.1.4.51	operator!=	67
5.1.4.52	operator!=	68
5.1.4.53	operator*	68
5.1.4.54	operator*	69
5.1.4.55	operator*	69
5.1.4.56	operator+	70
5.1.4.57	operator+	71
5.1.4.58	operator+	71
5.1.4.59	operator-	72
5.1.4.60	operator-	72
5.1.4.61	operator-	73
5.1.4.62	operator/	74
5.1.4.63	operator/	74
5.1.4.64	operator/	75
5.1.4.65	operator<<	76
5.1.4.66	operator==	76
5.1.4.67	operator==	77
5.1.4.68	operator==	77
5.1.4.69	operator>>	78
5.1.4.70	pow	78
5.1.4.71	pow	79
5.1.4.72	pow	79
5.1.4.73	sec	80

5.1.4.74	sech	80
5.1.4.75	sin	81
5.1.4.76	sinh	82
5.1.4.77	sqrt	84
5.1.4.78	sqrt	85
5.1.4.79	tan	85
5.1.4.80	tanh	87
5.1.5	Member Data Documentation	89
5.1.5.1	i	89
5.1.5.2	imaginary	89
5.1.5.3	real	90
5.2	DivisionByZeroException Class Reference	90
5.2.1	Detailed Description	90
5.2.2	Member Function Documentation	90
5.2.2.1	what	90
5.3	LogarithmOfZeroException Class Reference	91
5.3.1	Detailed Description	91
5.3.2	Member Function Documentation	91
5.3.2.1	what	91
5.4	ZeroToComplexPowerException Class Reference	91
5.4.1	Detailed Description	91
5.4.2	Member Function Documentation	92
5.4.2.1	what	92
5.5	ZeroToTheZerothPowerException Class Reference	92
5.5.1	Detailed Description	92
5.5.2	Member Function Documentation	92
5.5.2.1	what	92
6	File Documentation	93
6.1	include/ComplexMath.h File Reference	93
6.1.1	Detailed Description	94

6.1.2	Define Documentation	95
6.1.2.1	M_1_PII	95
6.1.2.2	M_2_PII	95
6.1.2.3	M_2_SQRTPII	95
6.1.2.4	M_EI	95
6.1.2.5	M_LN10I	95
6.1.2.6	M_LN2I	96
6.1.2.7	M_LOG10EI	96
6.1.2.8	M_LOG2EI	96
6.1.2.9	M_PI_2I	96
6.1.2.10	M_PI_4I	96
6.1.2.11	M_PII	97
6.1.2.12	M_SQRT1_2I	97
6.1.2.13	M_SQRT2I	97
6.2	include/ComplexNumber.h File Reference	97
6.2.1	Detailed Description	98
6.3	include/MathExceptions.h File Reference	98
6.3.1	Detailed Description	99
6.4	include/Namespace.h File Reference	99
6.4.1	Detailed Description	100
6.4.2	Define Documentation	100
6.4.2.1	BEGIN_NAMESPACE	100
6.4.2.2	END_NAMESPACE	100
6.5	src/ComplexMath.cpp File Reference	101
6.5.1	Detailed Description	104
6.5.2	Define Documentation	104
6.5.2.1	COMPLEX_MATH_CPP	104
6.5.3	Function Documentation	105
6.5.3.1	abs	105
6.5.3.2	abs2	105
6.5.3.3	acos	105

6.5.3.4	acos	105
6.5.3.5	acosh	105
6.5.3.6	acosh	105
6.5.3.7	acot	105
6.5.3.8	acot	105
6.5.3.9	acoth	106
6.5.3.10	acoth	106
6.5.3.11	acsc	106
6.5.3.12	acsc	106
6.5.3.13	acsch	106
6.5.3.14	acsch	106
6.5.3.15	arg	106
6.5.3.16	asec	106
6.5.3.17	asec	106
6.5.3.18	asech	107
6.5.3.19	asech	107
6.5.3.20	asin	107
6.5.3.21	asin	107
6.5.3.22	asinh	107
6.5.3.23	asinh	107
6.5.3.24	atan	107
6.5.3.25	atan	107
6.5.3.26	atanh	107
6.5.3.27	atanh	108
6.5.3.28	cbrt	108
6.5.3.29	conjugate	108
6.5.3.30	cos	108
6.5.3.31	cosh	108
6.5.3.32	cot	108
6.5.3.33	coth	108
6.5.3.34	csc	108

6.5.3.35	csch	109
6.5.3.36	exp	109
6.5.3.37	inverse	109
6.5.3.38	log	109
6.5.3.39	log	109
6.5.3.40	log10	109
6.5.3.41	log10	109
6.5.3.42	logabs	109
6.5.3.43	logb	110
6.5.3.44	logb	110
6.5.3.45	logb	110
6.5.3.46	logb	110
6.5.3.47	nthRoot	110
6.5.3.48	nthRoots	110
6.5.3.49	nthRoots	110
6.5.3.50	pow	110
6.5.3.51	pow	111
6.5.3.52	pow	111
6.5.3.53	sec	111
6.5.3.54	sech	111
6.5.3.55	sin	111
6.5.3.56	sinh	111
6.5.3.57	sqrt	111
6.5.3.58	sqrt	111
6.5.3.59	tan	112
6.5.3.60	tanh	112
6.6	src/ComplexNumber.cpp File Reference	112
6.6.1	Detailed Description	114
6.6.2	Define Documentation	114
6.6.2.1	COMPLEX_NUMBER_CPP	114
6.6.3	Function Documentation	114

6.6.3.1	operator!=	114
6.6.3.2	operator!=	114
6.6.3.3	operator!=	115
6.6.3.4	operator*	115
6.6.3.5	operator*	115
6.6.3.6	operator*	115
6.6.3.7	operator+	115
6.6.3.8	operator+	115
6.6.3.9	operator+	115
6.6.3.10	operator-	115
6.6.3.11	operator-	116
6.6.3.12	operator-	116
6.6.3.13	operator/	116
6.6.3.14	operator/	116
6.6.3.15	operator/	116
6.6.3.16	operator<<	116
6.6.3.17	operator==	117
6.6.3.18	operator==	117
6.6.3.19	operator==	117
6.6.3.20	operator>>	117

Chapter 1

Namespace Index

1.1 Namespace List

Here is a list of all namespaces with brief descriptions:

complex	Complex namespace	7
math	Math namespace	7

Chapter 2

Class Index

2.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

ComplexNumber	A class template which supports complex numbers	9
DivisionByZeroException	A division by zero exception	90
LogarithmOfZeroException	A logarithm of zero exception	91
ZeroToComplexPowerException	A zero to a complex power exception	91
ZeroToTheZerothPowerException	A zero to the zeroth power exception	92

Chapter 3

File Index

3.1 File List

Here is a list of all files with brief descriptions:

include/ComplexMath.h	Elementary complex mathematical functions	93
include/ComplexNumber.h	Representation of a complex number	97
include/MathExceptions.h	Classes that represent mathematical exceptions	98
include/Namespace.h	Defines namespace macros	99
src/ComplexMath.cpp	Elementary complex mathematical functions	101
src/ComplexNumber.cpp	Representation of a complex number	112

Chapter 4

Namespace Documentation

4.1 complex Namespace Reference

Complex namespace.

4.1.1 Detailed Description

Complex namespace. This is the complex namespace. It contains all classes and functions for dealing with complex numbers.

4.2 math Namespace Reference

Math namespace.

4.2.1 Detailed Description

Math namespace. This is the math namespace. It contains all classes and functions pertaining to mathematics.

Chapter 5

Class Documentation

5.1 ComplexNumber Class Reference

A class template which supports complex numbers.

```
#include <include/ComplexNumber.h>
```

Public Member Functions

- [ComplexNumber \(\)](#)
Default constructor for [ComplexNumber](#).
- [ComplexNumber \(const T &real, const T &imaginary\)](#)
Overloaded constructor for [ComplexNumber](#).
- [ComplexNumber \(const \[ComplexNumber\]\(#\)< T > &z\)](#)
Copy constructor for [ComplexNumber](#).
- virtual [~ComplexNumber \(\)](#)
Default destructor for [ComplexNumber](#).
- [ComplexNumber< T > operator+ \(\) const](#)
Unary plus operator.
- [ComplexNumber< T > operator- \(\) const](#)
Unary minus operator.
- template<class U >
[operator \[ComplexNumber\]\(#\)< U > \(\) const](#)
Type conversion operator.
- template<class U >
[ComplexNumber< T > & operator= \(const \[ComplexNumber\]\(#\)< U > &z\)](#)
Assigns a [ComplexNumber](#) to this [ComplexNumber](#).

- `template<class U >`
`ComplexNumber< T > & operator= (const U &real)`
Assigns a real number to this [ComplexNumber](#).
- `template<class U >`
`ComplexNumber< T > & operator+= (const ComplexNumber< U > &z)`
Adds and assigns a [ComplexNumber](#) to this.
- `template<class U >`
`ComplexNumber< T > & operator+= (const U &real)`
Adds and assigns a real value to this.
- `template<class U >`
`ComplexNumber< T > & operator-= (const ComplexNumber< U > &z)`
Subtracts and assigns a [ComplexNumber](#) to this.
- `template<class U >`
`ComplexNumber< T > & operator-= (const U &real)`
Subtracts and assigns a real value to this.
- `template<class U >`
`ComplexNumber< T > & operator*= (const ComplexNumber< U > &z)`
Multiplies and assigns a [ComplexNumber](#) to this.
- `template<class U >`
`ComplexNumber< T > & operator*= (const U &real)`
Multiplies and assigns a real value to this.
- `template<class U >`
`ComplexNumber< T > & operator/= (const ComplexNumber< U > &z)`
Divides and assigns a [ComplexNumber](#) to this.
- `template<class U >`
`ComplexNumber< T > & operator/= (const U &real)`
Divides and assigns a real value to this.
- `T getReal () const`
Get the real part.
- `T getImaginary () const`
Get the imaginary part.
- `void setReal (const T &real)`
Set the real part.
- `void setImaginary (const T &imaginary)`
Set the imaginary part.
- `template<>`
`const ComplexNumber< double > i (0, 1)`

Static Public Member Functions

- `static ComplexNumber< T > polar (const T &r, const T &theta)`
Construct a [ComplexNumber](#) from the given polar coordinates.

Static Public Attributes

- static const `ComplexNumber` < double > `i`
A public constant representing the value i .

Private Attributes

- T `real`
The value of the real part of the complex number.
- T `imaginary`
The value of the non-real part of the complex number.

Friends

- template<class U >
std::ostream & `operator<<` (std::ostream &out, const `ComplexNumber`< U > &z)
Inserts a `ComplexNumber` into the ostream.
- template<class U >
std::istream & `operator>>` (std::istream &in, `ComplexNumber`< U > &z)
Extracts a `ComplexNumber` from the istream.

Related Functions

(Note that these are not member functions.)

- template<class T >
T `abs` (const `ComplexNumber`< T > &z)
Absolute value of a `ComplexNumber`.
- template<class T >
T `abs2` (const `ComplexNumber`< T > &z)
Square absolute value of a `ComplexNumber`.
- template<class T >
T `arg` (const `ComplexNumber`< T > &z)
Argument of a `ComplexNumber`.
- template<class T >
T `logabs` (const `ComplexNumber`< T > &z)
Natural logarithm absolute value of a `ComplexNumber`.
- template<class T >
`ComplexNumber`< T > `conjugate` (const `ComplexNumber`< T > &z)

Complex conjugate of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > exp (const ComplexNumber< T > &z)`

Exponential function of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > inverse (const ComplexNumber< T > &z)`

Inverse of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > log (const ComplexNumber< T > &z)`

Natural logarithm of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > log (const T &x)`

Natural logarithm of a real value.

- `template<class T >`
`ComplexNumber< T > log10 (const ComplexNumber< T > &z)`

Logarithm base 10 of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > log10 (const T &x)`

Logarithm base 10 of a real value.

- `template<class T >`
`ComplexNumber< T > logb (const ComplexNumber< T > &z, const ComplexNumber< T > &b)`

Logarithm base b of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > logb (const ComplexNumber< T > &z, const T &b)`

Logarithm base b of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > logb (const T &x, const ComplexNumber< T > &b)`

Logarithm base b of a real value.

- `template<class T >`
`ComplexNumber< T > logb (const T &x, const T &b)`

Logarithm base b of a real value.

- `template<class T >`
`ComplexNumber< T > pow (const ComplexNumber< T > &a, const ComplexNumber< T > &b)`

Exponentiation of [ComplexNumbers](#).

- `template<class T >`
`ComplexNumber< T > pow (const ComplexNumber< T > &a, const T &b)`

Exponentiation of a [ComplexNumber](#) and real value.

- `template<class T >`
`ComplexNumber< T > pow (const T &a, const ComplexNumber< T > &b)`

Exponentiation of a real value and a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > sqrt (const ComplexNumber< T > &z)`
Square root of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > sqrt (const T &x)`
Square root of a real value.
- `template<class T >`
`ComplexNumber< T > cbrt (const ComplexNumber< T > &z)`
Cube root of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > nthRoot (const ComplexNumber< T > &z, const int n)`
 n th root of a [ComplexNumber](#)
- `template<class T >`
`std::vector< ComplexNumber< T > > nthRoots (const ComplexNumber< T > &z, const int n)`
 n th roots of a [ComplexNumber](#)
- `template<class T >`
`void nthRoots (const ComplexNumber< T > &z, std::vector< ComplexNumber< T > > roots, const int n)`
 n th roots of a [ComplexNumber](#)
- `template<class T >`
`ComplexNumber< T > sin (const ComplexNumber< T > &z)`
Sine of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > cos (const ComplexNumber< T > &z)`
Cosine of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > tan (const ComplexNumber< T > &z)`
Tangent of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > csc (const ComplexNumber< T > &z)`
Cosecant of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > sec (const ComplexNumber< T > &z)`
Secant of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > cot (const ComplexNumber< T > &z)`
Cotangent of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > asin (const ComplexNumber< T > &z)`
Arcsine of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > asin (const T &x)`
Arcsine of a real value.
- `template<class T >`
`ComplexNumber< T > acos (const ComplexNumber< T > &z)`
*Arccosine of a *ComplexNumber*.*
- `template<class T >`
`ComplexNumber< T > acos (const T &x)`
Arccosine of a real value.
- `template<class T >`
`ComplexNumber< T > atan (const ComplexNumber< T > &z)`
*Arctangent of a *ComplexNumber*.*
- `template<class T >`
`ComplexNumber< T > atan (const T &x)`
Arctangent of a real value.
- `template<class T >`
`ComplexNumber< T > acsc (const ComplexNumber< T > &z)`
*Arccosecant of a *ComplexNumber*.*
- `template<class T >`
`ComplexNumber< T > acsc (const T &x)`
Arccosecant of a real value.
- `template<class T >`
`ComplexNumber< T > asec (const ComplexNumber< T > &z)`
*Arcsecant of a *ComplexNumber*.*
- `template<class T >`
`ComplexNumber< T > asec (const T &x)`
Arcsecant of a real value.
- `template<class T >`
`ComplexNumber< T > acot (const ComplexNumber< T > &z)`
*Arccotangent of a *ComplexNumber*.*
- `template<class T >`
`ComplexNumber< T > acot (const T &x)`
Arccotangent of a real value.
- `template<class T >`
`ComplexNumber< T > sinh (const ComplexNumber< T > &z)`
*Hyperbolic sine of a *ComplexNumber*.*
- `template<class T >`
`ComplexNumber< T > cosh (const ComplexNumber< T > &z)`
*Hyperbolic cosine of a *ComplexNumber*.*
- `template<class T >`
`ComplexNumber< T > tanh (const ComplexNumber< T > &z)`

Hyperbolic tangent of a [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > csch` (const `ComplexNumber< T > &z`)
Hyperbolic cosecant of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > sech` (const `ComplexNumber< T > &z`)
Hyperbolic secant of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > coth` (const `ComplexNumber< T > &z`)
Hyperbolic cotangent of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > asinh` (const `ComplexNumber< T > &z`)
Hyperbolic arsine of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > asinh` (const `T &x`)
Hyperbolic arsine of a real value.
- `template<class T >`
`ComplexNumber< T > acosh` (const `ComplexNumber< T > &z`)
Hyperbolic arc cosine of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > acosh` (const `T &x`)
Hyperbolic arc cosine of a real value.
- `template<class T >`
`ComplexNumber< T > atanh` (const `ComplexNumber< T > &z`)
Hyperbolic artangent of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > atanh` (const `T &x`)
Hyperbolic artangent of a real value.
- `template<class T >`
`ComplexNumber< T > acsch` (const `ComplexNumber< T > &z`)
Hyperbolic arc cosecant of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > acsch` (const `T &x`)
Hyperbolic arc cosecant of a real value.
- `template<class T >`
`ComplexNumber< T > asech` (const `ComplexNumber< T > &z`)
Hyperbolic arc secant of a [ComplexNumber](#).
- `template<class T >`
`ComplexNumber< T > asech` (const `T &x`)
Hyperbolic arc secant of a real value.
- `template<class T >`
`ComplexNumber< T > acoth` (const `ComplexNumber< T > &z`)

Hyperbolic arcotangent of a [ComplexNumber](#).

- `template<class T >`
[ComplexNumber](#)< T > [acoth](#) (const T &x)
Hyperbolic arcotangent of a real value.
- `template<class T >`
[ComplexNumber](#)< T > [operator+](#) (const [ComplexNumber](#)< T > &lhs, const [ComplexNumber](#)< T > &rhs)
Adds a [ComplexNumber](#) to another [ComplexNumber](#).
- `template<class T >`
[ComplexNumber](#)< T > [operator+](#) (const [ComplexNumber](#)< T > &lhs, const T &rhs)
Adds a real value to a [ComplexNumber](#).
- `template<class T >`
[ComplexNumber](#)< T > [operator+](#) (const T &lhs, const [ComplexNumber](#)< T > &rhs)
Adds a [ComplexNumber](#) to a real value.
- `template<class T >`
[ComplexNumber](#)< T > [operator-](#) (const [ComplexNumber](#)< T > &lhs, const [ComplexNumber](#)< T > &rhs)
Subtracts a [ComplexNumber](#) from another [ComplexNumber](#).
- `template<class T >`
[ComplexNumber](#)< T > [operator-](#) (const [ComplexNumber](#)< T > &lhs, const T &rhs)
Subtracts a real value from a [ComplexNumber](#).
- `template<class T >`
[ComplexNumber](#)< T > [operator-](#) (const T &lhs, const [ComplexNumber](#)< T > &rhs)
Subtracts a [ComplexNumber](#) from a real value.
- `template<class T >`
[ComplexNumber](#)< T > [operator*](#) (const [ComplexNumber](#)< T > &lhs, const [ComplexNumber](#)< T > &rhs)
Multiplies a [ComplexNumber](#) by another [ComplexNumber](#).
- `template<class T >`
[ComplexNumber](#)< T > [operator*](#) (const [ComplexNumber](#)< T > &lhs, const T &rhs)
Multiplies a [ComplexNumber](#) by a real value.
- `template<class T >`
[ComplexNumber](#)< T > [operator*](#) (const T &lhs, const [ComplexNumber](#)< T > &rhs)
Multiplies a real value by a [ComplexNumber](#).
- `template<class T >`
[ComplexNumber](#)< T > [operator/](#) (const [ComplexNumber](#)< T > &lhs, const [ComplexNumber](#)< T > &rhs)

Divides a [ComplexNumber](#) by another [ComplexNumber](#).

- `template<class T >`
`ComplexNumber< T > operator/ (const ComplexNumber< T > &lhs, const T &rhs)`

Divides a [ComplexNumber](#) by a real value.

- `template<class T >`
`ComplexNumber< T > operator/ (const T &lhs, const ComplexNumber< T > &rhs)`

Divides a real value by a [ComplexNumber](#).

- `template<class T, class U >`
`bool operator== (const ComplexNumber< T > &lhs, const ComplexNumber< U > &rhs)`

Compares a [ComplexNumber](#) to another [ComplexNumber](#).

- `template<class T, class U >`
`bool operator== (const ComplexNumber< T > &lhs, const U &rhs)`

Compares a [ComplexNumber](#) to a real value.

- `template<class T, class U >`
`bool operator== (const T &lhs, const ComplexNumber< U > &rhs)`

Compares a real value to a [ComplexNumber](#).

- `template<class T, class U >`
`bool operator!= (const ComplexNumber< T > &lhs, const ComplexNumber< U > &rhs)`

Compares a [ComplexNumber](#) to another [ComplexNumber](#).

- `template<class T, class U >`
`bool operator!= (const ComplexNumber< T > &lhs, const U &rhs)`

Compares a [ComplexNumber](#) to a real value.

- `template<class T, class U >`
`bool operator!= (const T &lhs, const ComplexNumber< U > &rhs)`

Compares a real value to a [ComplexNumber](#).

5.1.1 Detailed Description

A class template which supports complex numbers.

This class template manages numbers on the complex plane, providing a set of operators and corresponding template functions

5.1.2 Constructor & Destructor Documentation

5.1.2.1 `ComplexNumber::ComplexNumber ()`

Default constructor for [ComplexNumber](#).

This constructor creates and initializes an empty [ComplexNumber](#), with default values of zero for the real and imaginary parts

Definition at line 74 of file ComplexNumber.cpp.

5.1.2.2 `ComplexNumber::ComplexNumber (const T & real, const T & imaginary)`

Overloaded constructor for [ComplexNumber](#).

This constructor creates and initializes a [ComplexNumber](#) with the specified real and imaginary values

Parameters

<code>in</code>	<code><i>real</i></code>	The value of the real component of this ComplexNumber
<code>in</code>	<code><i>imaginary</i></code>	The value of the non-real component of this Complex-Number

Definition at line 80 of file ComplexNumber.cpp.

5.1.2.3 `ComplexNumber::ComplexNumber (const ComplexNumber< T > & z)`

Copy constructor for [ComplexNumber](#).

This constructor creates and initializes a copy of a [ComplexNumber](#)

Parameters

<code>in</code>	<code>z</code>	The ComplexNumber to be copied
-----------------	----------------	--

Definition at line 86 of file ComplexNumber.cpp.

5.1.2.4 `ComplexNumber::~ComplexNumber ()` [virtual]

Default destructor for [ComplexNumber](#).

The destructor cleans up memory allocated by [ComplexNumber](#). [ComplexNumber](#) does not allocate any memory, and therefore, the destructor does nothing.

Definition at line 92 of file ComplexNumber.cpp.

5.1.3 Member Function Documentation

5.1.3.1 T ComplexNumber::getImaginary () const

Get the imaginary part.

Gets the non-real part of this [ComplexNumber](#)

Returns

The value of the non-real component of this [ComplexNumber](#)

Definition at line 229 of file ComplexNumber.cpp.

5.1.3.2 T ComplexNumber::getReal () const

Get the real part.

Gets the real part of this [ComplexNumber](#)

Returns

The value of the real component of this [ComplexNumber](#)

Definition at line 223 of file ComplexNumber.cpp.

5.1.3.3 template<> const ComplexNumber< double > ComplexNumber< double >::i (0, 1)

5.1.3.4 template<class U > ComplexNumber::operator ComplexNumber< U > () const

Type conversion operator.

Returns this [ComplexNumber](#) type-casted as a different [ComplexNumber](#)

Returns

This [ComplexNumber](#) type-casted as a different [ComplexNumber](#)

Definition at line 116 of file ComplexNumber.cpp.

5.1.3.5 template<class U > ComplexNumber< T > & ComplexNumber::operator*=(const ComplexNumber< U > & z)

Multiplies and assigns a [ComplexNumber](#) to this.

Multiplies this [ComplexNumber](#) by another [ComplexNumber](#) and assigns the product to this [ComplexNumber](#)

$$z = z \times z_1$$

Parameters

<code>in</code>	<code>z</code>	The ComplexNumber by which this ComplexNumber will be multiplied
-----------------	----------------	--

Returns

A reference to this [ComplexNumber](#) consisting of the product of this [ComplexNumber](#) and another [ComplexNumber](#)

Definition at line 178 of file `ComplexNumber.cpp`.

5.1.3.6 `template<class U > ComplexNumber< T > & ComplexNumber::operator*=(const U & real)`

Multiplies and assigns a real value to this.

Multiplies this [ComplexNumber](#) by a real value and assigns the product to this [ComplexNumber](#)

$$z = z \times x$$

Parameters

<code>in</code>	<code>real</code>	The real value by which this ComplexNumber will be multiplied
-----------------	-------------------	---

Returns

A reference to this [ComplexNumber](#) consisting of the product of this [ComplexNumber](#) and a real value

Definition at line 187 of file `ComplexNumber.cpp`.

5.1.3.7 `ComplexNumber< T > ComplexNumber::operator+() const`

Unary plus operator.

Returns the positive of this [ComplexNumber](#)

$$z_2 = +z_1 = a + bi$$

Returns

A [ComplexNumber](#) that is the positive of this [ComplexNumber](#)

Definition at line 104 of file ComplexNumber.cpp.

5.1.3.8 `template<class U > ComplexNumber< T > & ComplexNumber::operator+=(const ComplexNumber< U > & z)`

Adds and assigns a [ComplexNumber](#) to this.

Adds a [ComplexNumber](#) to this [ComplexNumber](#) and assigns the sum to this [ComplexNumber](#)

$$z = z + z_1$$

Parameters

in	z	The ComplexNumber which will be added to this ComplexNumber
----	---	---

Returns

A reference to this [ComplexNumber](#) consisting of the sum of this [ComplexNumber](#) and another [ComplexNumber](#)

Definition at line 144 of file ComplexNumber.cpp.

5.1.3.9 `template<class U > ComplexNumber< T > & ComplexNumber::operator+=(const U & real)`

Adds and assigns a real value to this.

Adds a real value to this [ComplexNumber](#) and assigns the sum to this [ComplexNumber](#)

$$z = z + x$$

Parameters

in	<i>real</i>	The real value which will be added to this ComplexNumber
----	-------------	--

Returns

A reference to this [ComplexNumber](#) consisting of the sum of this [ComplexNumber](#) and a real value

Definition at line 153 of file ComplexNumber.cpp.

5.1.3.10 `ComplexNumber< T > ComplexNumber::operator-() const`

Unary minus operator.

Finds the negative of this [ComplexNumber](#) and returns it

$$z_2 = -z_1 = -a_1 - b_1i$$

Returns

A [ComplexNumber](#) that is the negative of this [ComplexNumber](#)

Definition at line 110 of file ComplexNumber.cpp.

5.1.3.11 `template<class U > ComplexNumber< T > & ComplexNumber::operator-=(const ComplexNumber< U > & z)`

Subtracts and assigns a [ComplexNumber](#) to this.

Subtracts a [ComplexNumber](#) from this [ComplexNumber](#) and assigns the difference to this [ComplexNumber](#)

$$z = z - z_1$$

Parameters

in	z	The ComplexNumber which will be subtracted from this ComplexNumber
----	---	--

Returns

A reference to this [ComplexNumber](#) consisting of the difference of this [ComplexNumber](#) and another [ComplexNumber](#)

Definition at line 161 of file ComplexNumber.cpp.

5.1.3.12 `template<class U > ComplexNumber< T > & ComplexNumber::operator-= (const U & real)`

Subtracts and assigns a real value to this.

Subtracts a real value from this [ComplexNumber](#) and assigns the difference to this [ComplexNumber](#)

$$z = z - x$$

Parameters

<code>in</code>	<code>real</code>	The real value which will be subtracted from this ComplexNumber
-----------------	-------------------	---

Returns

A reference to this [ComplexNumber](#) consisting of the difference of this [ComplexNumber](#) and a real value

Definition at line 170 of file ComplexNumber.cpp.

5.1.3.13 `template<class U > ComplexNumber< T > & ComplexNumber::operator/= (const ComplexNumber< U > & z)`

Divides and assigns a [ComplexNumber](#) to this.

Divides this [ComplexNumber](#) by another [ComplexNumber](#) and assigns the quotient to this [ComplexNumber](#)

$$z = \frac{z}{z_1}$$

Parameters

<code>in</code>	<code>z</code>	The ComplexNumber by which this ComplexNumber will be divided
-----------------	----------------	---

Exceptions

DivisionByZeroException	if attempting to divide this ComplexNumber by a ComplexNumber <code>z</code> that is equal to zero, a DivisionByZeroException is thrown
---	---

Returns

A reference to this [ComplexNumber](#) consisting of the quotient of this [ComplexNumber](#) and another [ComplexNumber](#)

Definition at line 196 of file ComplexNumber.cpp.

5.1.3.14 `template<class U > ComplexNumber< T > & ComplexNumber::operator/= (const U & real)`

Divides and assigns a real value to this.

Divides this [ComplexNumber](#) by a real value and assigns the quotient to this [ComplexNumber](#)

$$z = \frac{z}{x}$$

Parameters

<i>in</i>	<i>real</i>	The real value by which this ComplexNumber will be divided
-----------	-------------	--

Exceptions

DivisionByZero-Exception	if attempting to divide this ComplexNumber by a value real that is equal to zero, a DivisionByZeroException is thrown
--	---

Returns

A reference to this [ComplexNumber](#) consisting of the quotient of this [ComplexNumber](#) and another [ComplexNumber](#)

Definition at line 211 of file ComplexNumber.cpp.

5.1.3.15 `template<class U > ComplexNumber< T > & ComplexNumber::operator= (const ComplexNumber< U > & z)`

Assigns a [ComplexNumber](#) to this [ComplexNumber](#).

Assigns a [ComplexNumber](#) to this [ComplexNumber](#), copying its private data

$$z = z_1$$

Parameters

in	z	The ComplexNumber which will be copied and assigned to this ComplexNumber
----	---	---

Returns

A reference to this [ComplexNumber](#) consisting of the copied contents of the [ComplexNumber](#)

Definition at line 123 of file ComplexNumber.cpp.

5.1.3.16 `template<class U > ComplexNumber< T > & ComplexNumber::operator= (const U & real)`

Assigns a real number to this [ComplexNumber](#).

Assigns a real number to this [ComplexNumber](#)

$$z = x$$

Parameters

in	<i>real</i>	The real value which will be copied and to this ComplexNumber
----	-------------	---

Returns

A reference to this [ComplexNumber](#) consisting of the copied contents of the real value

Definition at line 135 of file ComplexNumber.cpp.

5.1.3.17 `ComplexNumber< T > ComplexNumber::polar (const T & r, const T & theta)`
[static]

Construct a [ComplexNumber](#) from the given polar coordinates.

Converts the polar form of a complex number to the rectangular form used in this class

$$z = re^{i\varphi} = r(\cos \varphi + i \sin \varphi)$$

Parameters

in	<i>r</i>	The magnitude of the complex number
in	<i>theta</i>	The angle of the complex number

Returns

A [ComplexNumber](#)

Definition at line 98 of file ComplexNumber.cpp.

5.1.3.18 void ComplexNumber::setImaginary (const T & *imaginary*)

Set the imaginary part.

Sets the non-real part of this [ComplexNumber](#)

Parameters

in	<i>imaginary</i>	The value of the non-real component of this ComplexNumber
----	------------------	---

Definition at line 241 of file ComplexNumber.cpp.

5.1.3.19 void ComplexNumber::setReal (const T & *real*)

Set the real part.

Sets the real part of this [ComplexNumber](#)

Parameters

in	<i>real</i>	The value of the real component of this ComplexNumber
----	-------------	---

Definition at line 235 of file ComplexNumber.cpp.

5.1.4 Friends And Related Function Documentation

5.1.4.1 template<class T > T abs (const ComplexNumber< T > & z) [related]

Absolute value of a [ComplexNumber](#).

Computes the absolute value, magnitude, or modulus, of a [ComplexNumber](#)

$$r = |z| = \sqrt{a^2 + b^2}$$

Parameters

in	z	The ComplexNumber of which to compute its absolute value
----	---	--

Returns

The absolute value of a [ComplexNumber](#)

Definition at line 73 of file ComplexMath.cpp.

5.1.4.2 `template<class T> T abs2 (const ComplexNumber<T> & z)` [related]

Square absolute value of a [ComplexNumber](#).

Computes the square of the absolute value, magnitude or modulus, of a [ComplexNumber](#).

$$r^2 = |z|^2 = a^2 + b^2$$

Parameters

in	z	The ComplexNumber of which to compute its squared absolute value
----	---	--

Returns

The squared absolute value of a [ComplexNumber](#)

Definition at line 80 of file ComplexMath.cpp.

5.1.4.3 `template<class T> ComplexNumber<T> acos (const ComplexNumber<T> & z)` [related]

Arccosine of a [ComplexNumber](#).

Computes the complex arccosine, or inverse cosine, of a [ComplexNumber](#). This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\arccos z = -i \ln \left(z + i \sqrt{1 - z^2} \right)$$

This is based on the exponential definition of cosine

$$\theta = \arccos z \Rightarrow z = \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

So, to solve for θ , we form a quadratic equation

$$\begin{aligned} z &= \frac{e^{i\theta} + e^{-i\theta}}{2} \\ 2z &= e^{i\theta} + e^{-i\theta} \\ 2ze^{i\theta} &= e^{2i\theta} + 1 \\ 0 &= e^{2i\theta} - 2ze^{i\theta} + 1 \end{aligned}$$

Applying the quadratic formula,

$$e^{i\theta} = \frac{2z \pm \sqrt{(-2z)^2 - 4}}{2} = \frac{-2z \pm 2\sqrt{z^2 - 1}}{2} = z \pm \sqrt{z^2 - 1}$$

Choosing the positive branch,

$$\theta = \frac{\ln(z + \sqrt{z^2 - 1})}{i} = -i \ln(z + \sqrt{z^2 - 1})$$

Parameters

in	z	The ComplexNumber of which to compute the arccosine
----	---	---

Returns

An angle that is the arccosine of a [ComplexNumber](#)

Definition at line 361 of file ComplexMath.cpp.

5.1.4.4 `template<class T> ComplexNumber< T > acos (const T & x)` [related]

Arccosine of a real value.

Computes the complex arccosine, or inverse cosine, of a real value. This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\arccos x = -i \ln(x + i\sqrt{1 - x^2})$$

This is based on the exponential definition of cosine

$$\theta = \arccos x \Rightarrow x = \cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

So, to solve for θ , we form a quadratic equation

$$\begin{aligned}x &= \frac{e^{i\theta} + e^{-i\theta}}{2} \\2x &= e^{i\theta} + e^{-i\theta} \\2xe^{i\theta} &= e^{2i\theta} + 1 \\0 &= e^{2i\theta} - 2xe^{i\theta} + 1\end{aligned}$$

Applying the quadratic formula,

$$e^{i\theta} = \frac{2x \pm \sqrt{(-2x)^2 - 4}}{2} = \frac{-2x \pm 2\sqrt{x^2 - 1}}{2} = x \pm \sqrt{x^2 - 1}$$

Choosing the positive branch,

$$\theta = \frac{\ln(x + \sqrt{x^2 - 1})}{i} = -i \ln(x + \sqrt{x^2 - 1})$$

This can be further simplified to deal with the specific values of x that return complex values. Namely, where $\{x \in \mathbb{R} : |x| > 1\}$.

$$\begin{aligned}\arccos x &= -i \ln(x + i\sqrt{1 - x^2}) \\&= -i \ln(x - \sqrt{x^2 - 1}) \\&= -i \left(i\varphi + \ln|x - \sqrt{x^2 - 1}| \right) \\&= \varphi - i \ln|x - \sqrt{x^2 - 1}| \\&= \begin{cases} \pi - i \ln(-x + \sqrt{x^2 - 1}) & \text{if } x < -1 \\ 0 + i \ln\left(\frac{1}{x - \sqrt{x^2 - 1}}\right) & \text{if } x > 1 \end{cases} \\&= \begin{cases} \pi - i \operatorname{arcosh}(-x) & \text{if } x < -1 \\ i \ln\left(\frac{1}{x - \sqrt{x^2 - 1}} \left(\frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1} + x}\right)\right) & \text{if } x > 1 \end{cases} \\&= \begin{cases} \pi - i \operatorname{arcosh}(-x) & \text{if } x < -1 \\ i \ln(x + \sqrt{x^2 - 1}) & \text{if } x > 1 \end{cases} \\&= \begin{cases} \pi - i \operatorname{arcosh}(-x) & \text{if } x < -1 \\ i \operatorname{arcosh} x & \text{if } x > 1 \end{cases}\end{aligned}$$

Parameters

in	x	The real value of which to compute the arccosine
----	---	--

Returns

An angle that is the arccosine of a real value

Definition at line 371 of file ComplexMath.cpp.

5.1.4.5 `template<class T> ComplexNumber< T > acosh (const ComplexNumber< T > & z)` [related]

Hyperbolic arccosine of a [ComplexNumber](#).

Computes the complex hyperbolic arccosine, or inverse hyperbolic cosine, of a [Complex-Number](#). This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arcosh} z = \ln \left(z + \sqrt{z-1} \sqrt{z+1} \right)$$

This is based on the exponential definition of hyperbolic cosine

$$\alpha = \operatorname{arcosh} z \Rightarrow z = \cosh \alpha = \frac{e^\alpha + e^{-\alpha}}{2}$$

So, to solve for α , we form a quadratic equation

$$\begin{aligned} z &= \frac{e^\alpha + e^{-\alpha}}{2} \\ 2z &= e^\alpha + e^{-\alpha} \\ 2ze^\alpha &= e^{2\alpha} + 1 \\ 0 &= e^{2\alpha} - 2ze^\alpha + 1 \end{aligned}$$

Applying the quadratic formula,

$$e^\alpha = \frac{2z \pm \sqrt{(-2z)^2 - 4}}{2} = \frac{2z \pm 2\sqrt{z^2 - 1}}{2} = z \pm \sqrt{z^2 - 1} = z \pm \sqrt{z-1} \sqrt{z+1}$$

Choosing the positive branch,

$$\alpha = \ln \left(z + \sqrt{z-1} \sqrt{z+1} \right)$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic arcsine
----	---	--

Returns

An area that is the hyperbolic arcsine of a [ComplexNumber](#)

Definition at line 497 of file ComplexMath.cpp.

5.1.4.6 `template<class T> ComplexNumber< T > acosh (const T & x)` [related]

Hyperbolic arcsine of a real value.

Computes the complex hyperbolic arcsine, or inverse hyperbolic cosine, of a real value. This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arcosh} x = \ln \left(x + \sqrt{x^2 - 1} \right)$$

This is based on the exponential definition of hyperbolic cosine

$$\alpha = \operatorname{arcosh} x \Rightarrow x = \cosh \alpha = \frac{e^\alpha + e^{-\alpha}}{2}$$

So, to solve for α , we form a quadratic equation

$$\begin{aligned} x &= \frac{e^\alpha + e^{-\alpha}}{2} \\ 2x &= e^\alpha + e^{-\alpha} \\ 2xe^\alpha &= e^{2\alpha} + 1 \\ 0 &= e^{2\alpha} - 2xe^\alpha + 1 \end{aligned}$$

Applying the quadratic formula,

$$e^\alpha = \frac{2x \pm \sqrt{(-2x)^2 - 4}}{2} = \frac{2x \pm 2\sqrt{x^2 - 1}}{2} = x \pm \sqrt{x^2 - 1}$$

Choosing the positive branch,

$$\alpha = \ln \left(x + \sqrt{x^2 - 1} \right)$$

Parameters

in	x	The real value of which to compute the hyperbolic arcsine
----	---	---

Returns

An area that is the hyperbolic arcsine of a real value

Definition at line 507 of file ComplexMath.cpp.

5.1.4.7 `template<class T> ComplexNumber< T> acot (const ComplexNumber< T> & z)` [related]

Arccotangent of a [ComplexNumber](#).

Computes the complex arccotangent, or inverse cotangent, of a [ComplexNumber](#). This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\operatorname{arccot} z = \arctan \frac{1}{z} = \frac{i}{2} \ln \left(\frac{z-i}{z+i} \right)$$

This can be proven using the reciprocal identities as follows,

$$\theta = \operatorname{arccot} z \Rightarrow \cot \theta = z$$

$$\tan \theta = \frac{1}{z}$$

$$\theta = \arctan \frac{1}{z}$$

$$\operatorname{arccot} z = \arctan \frac{1}{z}$$

Parameters

in	z	The ComplexNumber of which to compute the arccotangent
----	---	--

Returns

An angle that is the arccotangent of a [ComplexNumber](#)

Definition at line 426 of file ComplexMath.cpp.

5.1.4.8 `template<class T> ComplexNumber<T> acot (const T & x)` [related]

Arccotangent of a real value.

Computes the complex arccotangent, or inverse cotangent, of a real value. This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

While the domain of the arccotangent function is \mathbb{R} , this function is included for the sake of completeness.

Parameters

in	x	The real value of which to compute the arccotangent
----	---	---

Returns

An angle that is the arccotangent of a real value

Definition at line 432 of file ComplexMath.cpp.

5.1.4.9 `template<class T> ComplexNumber<T> acoth (const ComplexNumber<T> & z)` [related]

Hyperbolic arccotangent of a [ComplexNumber](#).

Computes the complex hyperbolic arccotangent, or inverse hyperbolic cotangent, of a [ComplexNumber](#). This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arcoth} z = \operatorname{artanh} \frac{1}{z} = \frac{1}{2} \ln \left(\frac{z+1}{z-1} \right)$$

This can be proven using the reciprocal identities as follows,

$$\begin{aligned} \alpha = \operatorname{arcoth} z &\Rightarrow \operatorname{coth} \alpha = z \\ \tanh \alpha &= \frac{1}{z} \\ \alpha &= \operatorname{artanh} \frac{1}{z} \\ \operatorname{arcoth} z &= \operatorname{artanh} \frac{1}{z} \end{aligned}$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic arccotangent
----	---	---

Returns

An area that is the hyperbolic arcotangent of a [ComplexNumber](#)

Definition at line 569 of file ComplexMath.cpp.

5.1.4.10 `template<class T> ComplexNumber< T > acoth (const T & x)` [related]

Hyperbolic arcotangent of a real value.

Computes the complex hyperbolic arcotangent, or inverse hyperbolic cotangent, of a real value. This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arcoth} x = \operatorname{artanh} \frac{1}{x} = \frac{1}{2} \ln \left(\frac{x+1}{x-1} \right)$$

This can be proven using the reciprocal identities as follows,

$$\begin{aligned} \alpha = \operatorname{arcoth} x &\Rightarrow \operatorname{coth} \alpha = x \\ \tanh \alpha &= \frac{1}{x} \\ \alpha &= \operatorname{artanh} \frac{1}{x} \\ \operatorname{arcoth} x &= \operatorname{artanh} \frac{1}{x} \end{aligned}$$

Parameters

in	x	The real value of which to compute the hyperbolic arcotangent
----	---	---

Returns

An area that is the hyperbolic arcotangent of a real value

Definition at line 575 of file ComplexMath.cpp.

5.1.4.11 `template<class T> ComplexNumber< T > acsc (const ComplexNumber< T > & z)` [related]

Arccosecant of a [ComplexNumber](#).

Computes the complex arccosecant, or inverse cosecant, of a [ComplexNumber](#). This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\operatorname{arccsc} z = \arcsin \frac{1}{z} = -i \ln \left(\frac{i}{z} + \sqrt{1 - \frac{1}{z^2}} \right)$$

This can be proven using the reciprocal identities as follows,

$$\theta = \operatorname{arccsc} z \Rightarrow \csc \theta = z$$

$$\sin \theta = \frac{1}{z}$$

$$\theta = \arcsin \frac{1}{z}$$

$$\operatorname{arccsc} z = \arcsin \frac{1}{z}$$

Parameters

in	z	The ComplexNumber of which to compute the arccosecant
----	---	---

Returns

An angle that is the arccosecant of a [ComplexNumber](#)

Definition at line 402 of file ComplexMath.cpp.

5.1.4.12 `template<class T> ComplexNumber< T > acsc (const T & x)` [\[related\]](#)

Arccosecant of a real value.

Computes the complex arccosecant, or inverse cosecant, of a real value. This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\operatorname{arccsc} x = \arcsin \frac{1}{x} = -i \ln \left(\frac{i}{x} + \sqrt{1 - \frac{1}{x^2}} \right)$$

This can be proven using the reciprocal identities as follows,

$$\theta = \operatorname{arccsc} x \Rightarrow \csc \theta = x$$

$$\sin \theta = \frac{1}{x}$$

$$\theta = \arcsin \frac{1}{x}$$

$$\operatorname{arccsc} x = \arcsin \frac{1}{x}$$

Parameters

in	x	The real value of which to compute the arccosecant
----	---	--

Returns

An angle that is the arccosecant of a real value

Definition at line 408 of file ComplexMath.cpp.

5.1.4.13 `template<class T> ComplexNumber< T > acsch (const ComplexNumber< T > & z)` [related]

Hyperbolic arccosecant of a [ComplexNumber](#).

Computes the complex hyperbolic arccosecant, or inverse hyperbolic cosecant, of a [ComplexNumber](#). This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arsch} z = \operatorname{arsinh} \frac{1}{z} = \ln \left(\sqrt{1 + \frac{1}{z^2}} + \frac{1}{z} \right)$$

This can be proven using the reciprocal identities as follows,

$$\alpha = \operatorname{arsch} z \Rightarrow \operatorname{csch} \alpha = z$$

$$\sinh \alpha = \frac{1}{z}$$

$$\alpha = \operatorname{arsinh} \frac{1}{z}$$

$$\operatorname{arsch} z = \operatorname{arsinh} \frac{1}{z}$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic arcosecant
----	---	---

Returns

An area that is the hyperbolic arcosecant of a [ComplexNumber](#)

Definition at line 541 of file ComplexMath.cpp.

5.1.4.14 `template<class T> ComplexNumber< T > acsch (const T & x)` [related]

Hyperbolic arcosecant of a real value.

Computes the complex hyperbolic arcosecant, or inverse hyperbolic cosecant, of a real value. This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arsch} x = \operatorname{arsinh} \frac{1}{x} = \ln \left(\sqrt{1 + \frac{1}{x^2}} + \frac{1}{x} \right)$$

This can be proven using the reciprocal identities as follows,

$$\alpha = \operatorname{arsch} x \Rightarrow \operatorname{csch} \alpha = x$$

$$\sinh \alpha = \frac{1}{x}$$

$$\alpha = \operatorname{arsinh} \frac{1}{x}$$

$$\operatorname{arsch} x = \operatorname{arsinh} \frac{1}{x}$$

Parameters

in	x	The real value of which to compute the hyperbolic arcosecant
----	---	--

Returns

An area that is the hyperbolic arcosecant of a real value

Definition at line 547 of file ComplexMath.cpp.

5.1.4.15 `template<class T> T arg (const ComplexNumber< T > & z)` [related]

Argument of a [ComplexNumber](#).

Computes the argument, the angle between the positive real axis and the vector representing z .

$$\varphi = \arg z = \operatorname{atan2}(b, a)$$

Where

$$\operatorname{atan2}(y, x) = \begin{cases} \arctan\left(\frac{y}{x}\right) & \text{if } x > 0 \\ \pi + \arctan\left(\frac{y}{x}\right) & \text{if } y \geq 0, x < 0 \\ -\pi + \arctan\left(\frac{y}{x}\right) & \text{if } y < 0, x < 0 \\ \frac{\pi}{2} & \text{if } y > 0, x = 0 \\ -\frac{\pi}{2} & \text{if } y < 0, x = 0 \\ \text{undefined} & \text{if } y = 0, x = 0 \end{cases}$$

Parameters

in	z	The ComplexNumber of which to compute its argument
----	---	--

Returns

The argument of a [ComplexNumber](#)

Definition at line 87 of file ComplexMath.cpp.

5.1.4.16 `template<class T> ComplexNumber< T > asec (const ComplexNumber< T > & z)` [related]

Arcsecant of a [ComplexNumber](#).

Computes the complex arcsecant, or inverse secant, of a [ComplexNumber](#). This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\operatorname{arcsec} z = \arccos \frac{1}{z} = -i \ln \left(\frac{1}{z} + \sqrt{1 - \frac{i}{z^2}} \right)$$

This can be proven using the reciprocal identities as follows,

$$\begin{aligned}\theta = \operatorname{arcsec} z &\Rightarrow \sec \theta = z \\ \cos \theta &= \frac{1}{z} \\ \theta &= \arccos \frac{1}{z} \\ \operatorname{arcsec} z &= \arccos \frac{1}{z}\end{aligned}$$

Parameters

in	z	The ComplexNumber of which to compute the arcsecant
----	---	---

Returns

An angle that is the arcsecant of a [ComplexNumber](#)

Definition at line 414 of file ComplexMath.cpp.

5.1.4.17 `template<class T> ComplexNumber< T > asec (const T & x)` [\[related\]](#)

Arcsecant of a real value.

Computes the complex arcsecant, or inverse secant, of a real value. This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\operatorname{arcsec} x = \arccos \frac{1}{x} = -i \ln \left(\frac{1}{x} + \sqrt{1 - \frac{i}{x^2}} \right)$$

This can be proven using the reciprocal identities as follows,

$$\begin{aligned}\theta = \operatorname{arcsec} x &\Rightarrow \sec \theta = x \\ \cos \theta &= \frac{1}{x} \\ \theta &= \arccos \frac{1}{x} \\ \operatorname{arcsec} x &= \arccos \frac{1}{x}\end{aligned}$$

Parameters

in	x	The real value of which to compute the arcsecant
----	---	--

Returns

An angle that is the arcsecant of a real value

Definition at line 420 of file ComplexMath.cpp.

5.1.4.18 `template<class T> ComplexNumber< T > asech (const ComplexNumber< T > & z)` [related]

Hyperbolic arsecant of a [ComplexNumber](#).

Computes the complex hyperbolic arsecant, or inverse hyperbolic secant, of a [ComplexNumber](#). This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\alpha = \operatorname{arsech} z = \operatorname{arcosh} \frac{1}{z} = \ln \left(\sqrt{\frac{1}{z} - 1} \sqrt{\frac{1}{z} + 1} + \frac{1}{z} \right)$$

This can be proven using the reciprocal identities as follows,

$$\begin{aligned} \alpha = \operatorname{arsech} z &\Rightarrow \operatorname{sech} \alpha = z \\ \cosh \alpha &= \frac{1}{z} \\ \alpha &= \operatorname{arcosh} \frac{1}{z} \\ \operatorname{arsech} z &= \operatorname{arcosh} \frac{1}{z} \end{aligned}$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic arsecant
----	---	---

Returns

An area that is the hyperbolic arsecant of a [ComplexNumber](#)

Definition at line 553 of file ComplexMath.cpp.

5.1.4.19 `template<class T> ComplexNumber< T > asech (const T & x)` [related]

Hyperbolic arsecant of a real value.

Computes the complex hyperbolic arsecant, or inverse hyperbolic secant, of a real value. This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arsech} x = \operatorname{arcosh} \frac{1}{x} = \ln \left(\sqrt{\frac{1}{x} - 1} \sqrt{\frac{1}{x} + 1} + \frac{1}{x} \right)$$

This can be proven using the reciprocal identities as follows,

$$\alpha = \operatorname{arsech} x \Rightarrow \operatorname{sech} \alpha = x$$

$$\cosh \alpha = \frac{1}{x}$$

$$\alpha = \operatorname{arcosh} \frac{1}{x}$$

$$\operatorname{arsech} x = \operatorname{arcosh} \frac{1}{x}$$

Parameters

<code>in</code>	<code>x</code>	The real value of which to compute the hyperbolic arsecant
-----------------	----------------	--

Returns

An area that is the hyperbolic arsecant of a real value

Definition at line 559 of file ComplexMath.cpp.

5.1.4.20 `template<class T> ComplexNumber< T > asin (const ComplexNumber< T > & z)` [related]

Arcsine of a [ComplexNumber](#).

Computes the complex arcsine, or inverse sine, of a [ComplexNumber](#). This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\operatorname{arcsin} z = -i \ln \left(iz + \sqrt{1 - z^2} \right)$$

This is based on the exponential definition of sine

$$\theta = \arcsin z \Rightarrow z = \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

So, to solve for θ , we form a quadratic equation

$$\begin{aligned} z &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \\ 2iz &= e^{i\theta} - e^{-i\theta} \\ 2ize^{i\theta} &= e^{2i\theta} - 1 \\ 0 &= e^{2i\theta} - 2ize^{i\theta} - 1 \end{aligned}$$

Applying the quadratic formula,

$$e^{i\theta} = \frac{2iz \pm \sqrt{(2iz)^2 + 4}}{2} = \frac{2iz \pm 2\sqrt{1 - z^2}}{2} = iz \pm \sqrt{1 - z^2}$$

Choosing the positive branch,

$$\theta = \frac{\ln\left(iz + \sqrt{1 - z^2}\right)}{i} = -i \ln\left(iz + \sqrt{1 - z^2}\right)$$

Parameters

in	z	The ComplexNumber of which to compute the arcsine
----	---	---

Returns

An angle that is the arcsine of a [ComplexNumber](#)

Definition at line 336 of file ComplexMath.cpp.

5.1.4.21 `template<class T> ComplexNumber<T> asin (const T & x)` [[related](#)]

Arcsine of a real value.

Computes the complex arcsine, or inverse sine, of a real value. This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\arcsin x = -i \ln\left(ix + \sqrt{1 - x^2}\right)$$

This is based on the exponential definition of sine

$$\theta = \arcsin x \Rightarrow x = \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

So, to solve for θ , we form a quadratic equation

$$\begin{aligned} x &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \\ 2ix &= e^{i\theta} - e^{-i\theta} \\ 2ixe^{i\theta} &= e^{2i\theta} - 1 \\ 0 &= e^{2i\theta} - 2ixe^{i\theta} - 1 \end{aligned}$$

Applying the quadratic formula,

$$e^{i\theta} = \frac{2ix \pm \sqrt{(2ix)^2 + 4}}{2} = \frac{2ix \pm 2\sqrt{1-x^2}}{2} = ix \pm \sqrt{1-x^2}$$

Choosing the positive branch,

$$\theta = \frac{\ln\left(ix + \sqrt{1-x^2}\right)}{i} = -i \ln\left(ix + \sqrt{1-x^2}\right)$$

This can be further simplified to deal with the specific values of x that return complex values. Namely, where $\{x \in \mathbb{R} : |x| > 1\}$.

$$\begin{aligned}
\arcsin x &= -i \ln \left(ix + \sqrt{1-x^2} \right) \\
&= -i \ln \left(i \left(x + \sqrt{x^2-1} \right) \right) \\
&= -i \left(i\varphi + \ln \left| i \left(x + \sqrt{x^2-1} \right) \right| \right) \\
&= \varphi - i \ln \left| x + \sqrt{x^2-1} \right| \\
&= \begin{cases} -\frac{\pi}{2} + i \ln \left(\frac{1}{-x - \sqrt{x^2-1}} \right) & \text{if } x < -1 \\ \frac{\pi}{2} - i \ln \left(x + \sqrt{x^2-1} \right) & \text{if } x > 1 \end{cases} \\
&= \begin{cases} -\frac{\pi}{2} + i \ln \left(\frac{1}{-x - \sqrt{x^2-1}} \left(\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}-x} \right) \right) & \text{if } x < -1 \\ \frac{\pi}{2} - i \operatorname{arcosh} x & \text{if } x > 1 \end{cases} \\
&= \begin{cases} -\frac{\pi}{2} + i \ln \left(-x + \sqrt{x^2-1} \right) & \text{if } x < -1 \\ \frac{\pi}{2} - i \operatorname{arcosh} x & \text{if } x > 1 \end{cases} \\
&= \begin{cases} -\frac{\pi}{2} + i \operatorname{arcosh}(-x) & \text{if } x < -1 \\ \frac{\pi}{2} - i \operatorname{arcosh} x & \text{if } x > 1 \end{cases}
\end{aligned}$$

Parameters

in	x	The real value of which to compute the arcsine
----	---	--

Returns

An angle that is the arcsine of a real value

Definition at line 347 of file ComplexMath.cpp.

5.1.4.22 `template<class T> ComplexNumber< T > asinh (const ComplexNumber< T > & z)` [related]

Hyperbolic arsine of a [ComplexNumber](#).

Computes the complex hyperbolic arsine, or inverse hyperbolic sine, of a [Complex-Number](#). This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arsinh} z = \ln \left(z + \sqrt{z^2 + 1} \right)$$

This is based on the definition of hyperbolic sine

$$\alpha = \operatorname{arsinh} z \Rightarrow z = \sinh \alpha = \frac{e^\alpha - e^{-\alpha}}{2}$$

So, to solve for α , we form a quadratic equation

$$\begin{aligned} z &= \frac{e^\alpha - e^{-\alpha}}{2} \\ 2z &= e^\alpha - e^{-\alpha} \\ 2ze^\alpha &= e^{2\alpha} - 1 \\ 0 &= e^{2\alpha} - 2ze^\alpha - 1 \end{aligned}$$

Applying the quadratic formula,

$$e^\alpha = \frac{2z \pm \sqrt{(-2z)^2 + 4}}{2} = \frac{2z \pm 2\sqrt{z^2 + 1}}{2} = z \pm \sqrt{z^2 + 1}$$

Choosing the positive branch,

$$\alpha = \ln \left(z + \sqrt{z^2 + 1} \right)$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic arsine
----	-----	---

Returns

An area that is the hyperbolic arsine of a [ComplexNumber](#)

Definition at line 481 of file ComplexMath.cpp.

5.1.4.23 `template<class T> ComplexNumber< T > asinh (const T & x)` [\[related\]](#)

Hyperbolic arsine of a real value.

Computes the complex hyperbolic arsine, or inverse hyperbolic sine, of a real value. This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{arsinh} x = \ln \left(x + \sqrt{x^2 + 1} \right)$$

This is based on the definition of hyperbolic sine

$$\alpha = \operatorname{arsinh} x \Rightarrow x = \sinh \alpha = \frac{e^\alpha - e^{-\alpha}}{2}$$

So, to solve for α , we form a quadratic equation

$$\begin{aligned} x &= \frac{e^\alpha - e^{-\alpha}}{2} \\ 2x &= e^\alpha - e^{-\alpha} \\ 2xe^\alpha &= e^{2\alpha} - 1 \\ 0 &= e^{2\alpha} - 2xe^\alpha - 1 \end{aligned}$$

Applying the quadratic formula,

$$e^\alpha = \frac{2x \pm \sqrt{(-2x)^2 + 4}}{2} = \frac{2x \pm 2\sqrt{x^2 + 1}}{2} = x \pm \sqrt{x^2 + 1}$$

Choosing the positive branch,

$$\alpha = \ln \left(x + \sqrt{x^2 + 1} \right)$$

Parameters

in	x	The real value of which to compute the hyperbolic arsine
----	---	--

Returns

An area that is the hyperbolic arsine of a real value

Definition at line 491 of file ComplexMath.cpp.

5.1.4.24 `template<class T> ComplexNumber<T> atan (const ComplexNumber< T
> &z) [related]`

Arctangent of a [ComplexNumber](#).

Computes the complex arctangent, or inverse tangent, of a [ComplexNumber](#). This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

$$\arctan z = \frac{i}{2} \ln \left(\frac{i+z}{i-z} \right)$$

This is based on the exponential definition of tangent

$$\theta = \arctan z \Rightarrow z = \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{2i}e^{i\theta} - e^{-i\theta}}{\frac{1}{2}e^{i\theta} + e^{-i\theta}} = \frac{e^{i\theta} - e^{-i\theta}}{i(e^{i\theta} + e^{-i\theta})}$$

So, to solve for θ , we form a quadratic equation

$$\begin{aligned} z &= \frac{e^{i\theta} - e^{-i\theta}}{i(e^{i\theta} + e^{-i\theta})} \\ iz(e^{i\theta} + e^{-i\theta}) &= e^{i\theta} - e^{-i\theta} \\ iz e^{i\theta} + iz e^{-i\theta} &= e^{i\theta} - e^{-i\theta} \\ iz e^{2i\theta} + iz &= e^{2i\theta} - 1 \\ (iz - 1)e^{2i\theta} &= -(iz + 1) \\ e^{2i\theta} &= -\frac{iz + 1}{iz - 1} = \frac{i - z}{i + z} \end{aligned}$$

Choosing the positive branch,

$$\theta = \frac{\ln\left(\sqrt{\frac{i-z}{i+z}}\right)}{i} = -i \ln\left(\left(\frac{i+z}{i-z}\right)^{-\frac{1}{2}}\right) = \frac{i}{2} \ln\left(\frac{i+z}{i-z}\right)$$

Parameters

in	z	The ComplexNumber of which to compute the arctangent
----	---	--

Returns

An angle that is the arctangent of a [ComplexNumber](#)

Definition at line 385 of file ComplexMath.cpp.

5.1.4.25 `template<class T> ComplexNumber< T > atan (const T & x)` [\[related\]](#)

Arctangent of a real value.

Computes the complex arctangent, or inverse tangent, of a real value. This value represents an arclength of a sector of the unit circle $x^2 + y^2 = 1$.

While the domain of the arctangent function is \mathbb{R} , this function is included for the sake of completeness.

Parameters

in	x	The real value of which to compute the arctangent
----	---	---

Returns

An angle that is the arctangent of a real value

Definition at line 396 of file ComplexMath.cpp.

5.1.4.26 `template<class T> ComplexNumber< T > atanh (const ComplexNumber< T > & z)` [related]

Hyperbolic artangent of a [ComplexNumber](#).

Computes the complex hyperbolic artangent, or inverse hyperbolic tangent, of a [ComplexNumber](#). This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{artanh} z = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right)$$

This is based on the exponential definition of hyperbolic tangent

$$\alpha = \operatorname{artanh} z \Rightarrow z = \tanh \alpha = \frac{\sinh \alpha}{\cosh \alpha} = \frac{\frac{1}{2}e^{\alpha} - e^{-\alpha}}{\frac{1}{2}e^{\alpha} + e^{-\alpha}} = \frac{e^{\alpha} - e^{-\alpha}}{e^{\alpha} + e^{-\alpha}}$$

So, to solve for α , we form a quadratic equation

$$\begin{aligned} z &= \frac{e^{\alpha} - e^{-\alpha}}{e^{\alpha} + e^{-\alpha}} \\ z(e^{\alpha} + e^{-\alpha}) &= e^{\alpha} - e^{-\alpha} \\ ze^{\alpha} + ze^{-\alpha} &= e^{\alpha} - e^{-\alpha} \\ ze^{2\alpha} + z &= e^{2\alpha} - 1 \\ (z-1)e^{2\alpha} &= -(z+1) \\ e^{2\alpha} &= -\frac{z+1}{z-1} = \frac{1+z}{1-z} \end{aligned}$$

Choosing the positive branch,

$$\alpha = \ln \left(\sqrt{\frac{1+z}{1-z}} \right) = \frac{1}{2} \ln \left(\frac{1+z}{1-z} \right)$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic artangent
----	---	--

Returns

An area that is the hyperbolic artangent of a [ComplexNumber](#)

Definition at line 521 of file ComplexMath.cpp.

5.1.4.27 `template<class T> ComplexNumber< T > atanh (const T & x)` [\[related\]](#)

Hyperbolic artangent of a real value.

Computes the complex hyperbolic artangent, or inverse hyperbolic tangent, of a real value. This value represents an area of a sector of the unit hyperbola $x^2 - y^2 = 1$.

$$\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$

This is based on the exponential definition of hyperbolic tangent

$$\alpha = \operatorname{artanh} x \Rightarrow x = \tanh \alpha = \frac{\sinh \alpha}{\cosh \alpha} = \frac{\frac{1}{2}e^{\alpha} - e^{-\alpha}}{\frac{1}{2}e^{\alpha} + e^{-\alpha}} = \frac{e^{\alpha} - e^{-\alpha}}{e^{\alpha} + e^{-\alpha}}$$

So, to solve for α , we form a quadratic equation

$$\begin{aligned} x &= \frac{e^{\alpha} - e^{-\alpha}}{e^{\alpha} + e^{-\alpha}} \\ x(e^{\alpha} + e^{-\alpha}) &= e^{\alpha} - e^{-\alpha} \\ xe^{\alpha} + xe^{-\alpha} &= e^{\alpha} - e^{-\alpha} \\ xe^{2\alpha} + x &= e^{2\alpha} - 1 \\ (x-1)e^{2\alpha} &= -(x+1) \\ e^{2\alpha} &= -\frac{x+1}{x-1} = \frac{1+x}{1-x} \end{aligned}$$

Choosing the positive branch,

$$\alpha = \ln \left(\sqrt{\frac{1+x}{1-x}} \right) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$

Parameters

in	x	The real value of which to compute the hyperbolic artangent
----	---	---

Returns

An area that is the hyperbolic artangent of a real value

Definition at line 531 of file ComplexMath.cpp.

5.1.4.28 `template<class T> ComplexNumber< T > cbrt (const ComplexNumber< T > & z)` [related]

Cube root of a [ComplexNumber](#).

Computes the principle cube root of a [ComplexNumber](#).

If $z = re^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$, then

$$\sqrt[3]{z} = \sqrt[3]{r}e^{i\frac{\varphi}{3}} = (a^2 + b^2)^{\frac{1}{6}} \left(\cos\left(\frac{\varphi}{3}\right) + i \sin\left(\frac{\varphi}{3}\right) \right)$$

This is a result of de Moivre's formula,

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$

which can be proven by Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

and the rules of exponentiation,

$$(e^{ix})^n = e^{inx}$$

So,

$$e^{i(nx)} = (\cos x + i \sin x)^n = \cos nx + i \sin nx$$

Parameters

in	z	The ComplexNumber of which to compute the cube root
----	---	---

Returns

A [ComplexNumber](#) that is the cube root of a [ComplexNumber](#)

Definition at line 242 of file ComplexMath.cpp.

5.1.4.29 `template<class T > ComplexNumber< T > conjugate (const ComplexNumber< T > & z)` [related]

Complex conjugate of a [ComplexNumber](#).

Finds the complex conjugate of a [ComplexNumber](#)

$$\bar{z} = a - bi$$

Parameters

in	z	The ComplexNumber of which to find the complex conjugate.
----	---	---

Returns

A [ComplexNumber](#) that is the complex conjugate of another [ComplexNumber](#)

Definition at line 105 of file ComplexMath.cpp.

5.1.4.30 `template<class T > ComplexNumber< T > cos (const ComplexNumber< T > & z)` [related]

Cosine of a [ComplexNumber](#).

Computes the complex cosine of a [ComplexNumber](#).

$$\cos z = \cos(a + bi) = \cos a \cos(bi) - \sin a \sin(bi) = \cos a \cosh b - i \sin a \sinh b$$

This is a result of the addition or subtraction theorems or formulae, which can be proven by Euler's formula, $e^{ix} = \cos x + i \sin x$, and by noting the symmetry of the sine and cosine functions as follows,

$$\begin{aligned} \cos(\alpha \pm \beta) + i \sin(\alpha \pm \beta) &= e^{i(\alpha \pm \beta)} \\ &= e^{i\alpha} e^{\pm i\beta} \\ &= (\cos \alpha + i \sin \alpha)(\cos \beta \pm i \sin \beta) \\ &= (\cos \alpha \cos \beta \mp \sin \alpha \sin \beta) + i(\sin \alpha \cos \beta \pm \cos \alpha \sin \beta) \end{aligned}$$

Thus,

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta\end{aligned}$$

Since, by the exponential definitions of the trigonometric functions derived from Euler's formula and the definitions of the hyperbolic functions,

$$\begin{aligned}\sin x &= \frac{e^{ix} - e^{-ix}}{2i} \\ \cos x &= \frac{e^{ix} + e^{-ix}}{2} \\ \sinh x &= \frac{e^x - e^{-x}}{2} \\ \cosh x &= \frac{e^x + e^{-x}}{2}\end{aligned}$$

Then,

$$\begin{aligned}\sin(ix) &= \frac{e^{i^2x} - e^{-i^2x}}{2i} = \frac{e^{-x} - e^x}{2i} = \frac{e^x - e^{-x}}{2} i = i \sinh x \\ \cos(ix) &= \frac{e^{i^2x} + e^{-i^2x}}{2} = \frac{e^{-x} + e^x}{2} = \cosh x\end{aligned}$$

Parameters

in	z	The ComplexNumber of which to compute the cosine
----	---	--

Returns

A [ComplexNumber](#) that is the cosine of a [ComplexNumber](#)

Definition at line 300 of file ComplexMath.cpp.

5.1.4.31 `template<class T> ComplexNumber<T> cosh (const ComplexNumber<T> & z)` [\[related\]](#)

Hyperbolic cosine of a [ComplexNumber](#).

Computes the complex hyperbolic cosine of a [ComplexNumber](#).

$$\cosh z = \cosh(a + bi) = \cosh a \cos(bi) + \sinh a \sin(bi) = \cosh a \cos b + i \sinh a \sin b$$

This is a result of the addition or subtraction theorems or formulae,

$$\begin{aligned}\sinh(x \pm y) &= \sinh x \cosh y \pm \cosh x \sinh y \\ \cosh(x \pm y) &= \cosh x \cosh y \pm \sinh x \sinh y\end{aligned}$$

which can be proven as follows, also noting the symmetry of the hyperbolic sine and hyperbolic cosine functions,

$$\begin{aligned}\sinh(x + y) &= \frac{e^{x+y} - e^{-x-y}}{2} \\ &= \frac{e^x e^y - e^x e^{-y} + e^x e^{-y} - e^{-x} e^{-y}}{2} \\ &= e^x \left(\frac{e^y - e^{-y}}{2} \right) + e^{-y} \left(\frac{e^x - e^{-x}}{2} \right) \\ &= (\cosh x + \sinh x) \sinh y + (\cosh y - \sinh y) \sinh x \\ &= \cosh x \sinh y + \sinh x \sinh y + \sinh x \cosh y - \sinh x \sinh y \\ &= \sinh x \cosh y + \cosh x \sinh y\end{aligned}$$

$$\begin{aligned}\cosh(x + y) &= \frac{e^{x+y} + e^{-x-y}}{2} \\ &= \frac{e^x e^y - e^x e^{-y} + e^x e^{-y} + e^{-x} e^{-y}}{2} \\ &= e^x \left(\frac{e^y - e^{-y}}{2} \right) + e^{-y} \left(\frac{e^x + e^{-x}}{2} \right) \\ &= (\cosh x + \sinh x) \sinh y + (\cosh y - \sinh y) \cosh x \\ &= \cosh x \sinh y + \sinh x \sinh y + \cosh x \cosh y - \cosh x \sinh y \\ &= \cosh x \cosh y + \sinh x \sinh y\end{aligned}$$

Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

Since, by the exponential definitions of the trigonometric functions derived from Euler's formula and the definitions of the hyperbolic functions,

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Then,

$$\sinh(xi) = \frac{e^{xi} - e^{-xi}}{2} = i \sin x$$

$$\cosh(xi) = \frac{e^{xi} + e^{-xi}}{2} = \cos x$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic cosine
----	---	---

Returns

A [ComplexNumber](#) that is the hyperbolic cosine of a [ComplexNumber](#)

Definition at line 445 of file ComplexMath.cpp.

5.1.4.32 `template<class T> ComplexNumber< T > cot (const ComplexNumber< T > & z)` [related]

Cotangent of a [ComplexNumber](#).

Computes the complex cotangent of a [ComplexNumber](#).

$$\cot z = \frac{1}{\tan z}$$

Parameters

in	z	The ComplexNumber of which to compute the cotangent
----	---	---

Returns

A [ComplexNumber](#) that is the cotangent of a [ComplexNumber](#)

Definition at line 330 of file ComplexMath.cpp.

5.1.4.33 `template<class T> ComplexNumber< T > coth (const ComplexNumber< T > & z)` [related]

Hyperbolic cotangent of a [ComplexNumber](#).

Computes the complex hyperbolic cotangent of a [ComplexNumber](#).

$$\coth z = \frac{1}{\tanh z}$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic cotangent
----	---	--

Returns

A [ComplexNumber](#) that is the hyperbolic cotangent of a [ComplexNumber](#)

Definition at line 475 of file ComplexMath.cpp.

5.1.4.34 `template<class T> ComplexNumber< T > csc (const ComplexNumber< T > & z)` [related]

Cosecant of a [ComplexNumber](#).

Computes the complex cosecant of a [ComplexNumber](#).

$$\csc z = \frac{1}{\sin z}$$

Parameters

in	z	The ComplexNumber of which to compute the cosecant
----	---	--

Returns

A [ComplexNumber](#) that is the cosecant of a [ComplexNumber](#)

Definition at line 318 of file ComplexMath.cpp.

5.1.4.35 `template<class T> ComplexNumber< T > csch (const ComplexNumber< T > & z)` [related]

Hyperbolic cosecant of a [ComplexNumber](#).

Computes the complex hyperbolic cosecant of a [ComplexNumber](#).

$$\operatorname{csch} z = \frac{1}{\sinh z}$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic cosecant
----	---	---

Returns

A [ComplexNumber](#) that is the hyperbolic cosecant of a [ComplexNumber](#)

Definition at line 463 of file ComplexMath.cpp.

5.1.4.36 `template<class T> ComplexNumber< T > exp (const ComplexNumber< T > & z)` [related]

Exponential function of a [ComplexNumber](#).

Computes the e raised to a [ComplexNumber](#).

$$\exp(z) = e^z = e^{a+bi} = e^a e^{bi} = e^a (\cos b + i \sin b) = e^a \cos b + i e^a \sin b$$

This comes from Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

Parameters

in	z	The ComplexNumber to which e is raised
----	---	--

Returns

A [ComplexNumber](#) that is the exponent of a [ComplexNumber](#)

Definition at line 111 of file ComplexMath.cpp.

5.1.4.37 `template<class T> ComplexNumber< T > inverse (const ComplexNumber< T > & z)` [related]

Inverse of a [ComplexNumber](#).

Finds the inverse, or reciprocal, of a [ComplexNumber](#).

$$z^{-1} = \frac{1}{z} = \frac{1}{a+bi} = \frac{1}{a+bi} \frac{a-bi}{a-bi} = \frac{a-bi}{a^2+b^2} = \frac{\bar{z}}{|z|^2}$$

Parameters

in	z	The ComplexNumber of which the inverse is to be found
----	---	---

Returns

A [ComplexNumber](#) that is the inverse of a [ComplexNumber](#)

Definition at line 118 of file ComplexMath.cpp.

5.1.4.38 `template<class T> ComplexNumber< T > log (const ComplexNumber< T > & z)` [related]

Natural logarithm of a [ComplexNumber](#).

Computes the natural logarithm of a [ComplexNumber](#).

If $z = re^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$, then

$$\ln z = \ln (re^{i\varphi}) = \ln r + i\varphi$$

Parameters

in	z	The ComplexNumber of which the natural logarithm is to be found
----	---	---

Returns

A [ComplexNumber](#) that is the natural logarithm of a [ComplexNumber](#)

Definition at line 128 of file ComplexMath.cpp.

5.1.4.39 `template<class T> ComplexNumber< T > log (const T & x)` [related]

Natural logarithm of a real value.

Computes the natural logarithm of a real value. This is useful for when $x < 0$.

If $x = re^{i\varphi}$, where $r = |x|$ and $\varphi = \arg x$, then

$$\ln x = \ln(re^{i\varphi}) = \ln r + i\varphi$$

For values $x > 0$, $\arg x = 0$, and for values $x < 0$, $\arg x = -\pi$.

Parameters

in	x	The real value of which the natural logarithm is to be found
----	---	--

Returns

A [ComplexNumber](#) that is the natural logarithm of a real value

Definition at line 137 of file ComplexMath.cpp.

5.1.4.40 `template<class T> ComplexNumber< T > log10 (const ComplexNumber< T > & z)` [related]

Logarithm base 10 of a [ComplexNumber](#).

Computes the base 10 logarithm of a [ComplexNumber](#).

If $z = re^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$, then

$$\log_{10} z = \log_{10}(re^{i\varphi}) = \frac{\ln r + i\varphi}{\ln 10} = \frac{\ln z}{\ln 10}$$

This uses the change of base formula, which says

$$\log_b x = \frac{\log_k x}{\log_k b}$$

By the definition of a logarithm,

$$x = b^a \iff \log_b x = a$$

and using the base k logarithm,

$$\log_k x = \log_k b^a = a \log_k b \iff \frac{\log_k x}{\log_k b} = a$$

Since the natural logarithm is defined by the standard C library, and the desired base is 10, we use $k = e$ and $b = 10$.

Parameters

in	z	The ComplexNumber of which the natural logarithm is to be found
----	---	---

Returns

A [ComplexNumber](#) that is the base 10 logarithm of a [ComplexNumber](#)

Definition at line 146 of file ComplexMath.cpp.

5.1.4.41 `template<class T> ComplexNumber< T > log10(const T & x)` [related]

Logarithm base 10 of a real value.

Computes the base 10 logarithm of a real value. This is useful for when $x < 0$.

If $x = re^{i\varphi}$, where $r = |x|$ and $\varphi = \arg x$, then

$$\log_{10} z = \log_{10}(re^{i\varphi}) = \frac{\ln r + i\varphi}{\ln 10} = \frac{\ln z}{\ln 10}$$

This uses the change of base formula, which says

$$\log_b x = \frac{\log_k x}{\log_k b}$$

By the definition of a logarithm,

$$x = b^a \iff \log_b x = a$$

and using the base k logarithm,

$$\log_k x = \log_k b^a = a \log_k b \iff \frac{\log_k x}{\log_k b} = a$$

Since the natural logarithm is defined by the standard C library, and the desired base is 10, we use $k = e$ and $b = 10$.

Parameters

in	x	The real value of which the natural logarithm is to be found
----	---	--

Returns

A [ComplexNumber](#) that is the natural logarithm of a real value

Definition at line 152 of file ComplexMath.cpp.

5.1.4.42 `template<class T > T logabs (const ComplexNumber< T > & z)`
[related]

Natural logarithm absolute value of a [ComplexNumber](#).

Computes the natural logarithm of the absolute value, magnitude, or modulus of a [ComplexNumber](#).

$$x = \ln |z| = \ln \left(\sqrt{a^2 + b^2} \right) = \frac{1}{2} \ln (a^2 + b^2) = \frac{1}{2} \ln |z|^2$$

Parameters

in	z	The ComplexNumber of which to compute the natural logarithm of its absolute value
----	---	---

Returns

The argument of a [ComplexNumber](#)

Definition at line 96 of file ComplexMath.cpp.

5.1.4.43 `template<class T > ComplexNumber< T > logb (const ComplexNumber< T > & z, const ComplexNumber< T > & b)` [related]

Logarithm base b of a [ComplexNumber](#).

Computes the complex base b logarithm of a [ComplexNumber](#).

If $z = r_a e^{i\varphi_a}$ and $b = r_b e^{i\varphi_b}$, where $r_a = |z|$ and $\varphi_a = \arg z$ and $r_b = |b|$ and $\varphi_b = \arg b$, then

$$\log_b z = \log_b(r_a e^{i\varphi_a}) = \frac{\ln r_a + i\varphi_a}{\ln b} = \frac{\ln r_a + i\varphi_a}{\ln r_b + i\varphi_b} = \frac{\ln z}{\ln b}$$

This uses the change of base formula, which says

$$\log_b x = \frac{\log_k x}{\log_k b}$$

By the definition of a logarithm,

$$x = b^a \iff \log_b x = a$$

and using the base k logarithm,

$$\log_k x = \log_k b^a = a \log_k b \iff \frac{\log_k x}{\log_k b} = a$$

Since the natural logarithm is defined by the standard C library, and the desired base is b , we use $k = e$ and $b = b$.

Parameters

in	z	The ComplexNumber of which the logarithm base b is to be found
in	b	The ComplexNumber that is the base of the logarithm

Returns

A [ComplexNumber](#) that is the base b logarithm of a [ComplexNumber](#)

Definition at line 162 of file ComplexMath.cpp.

5.1.4.44 `template<class T> ComplexNumber< T > logb (const ComplexNumber< T > & z, const T & b)` [\[related\]](#)

Logarithm base b of a [ComplexNumber](#).

Computes the real base b logarithm of a [ComplexNumber](#). This is useful for when $b < 0$.

If $z = r_a e^{i\varphi_a}$ and $b = r_b e^{i\varphi_b}$, where $r_a = |z|$ and $\varphi_a = \arg z$ and $r_b = |b|$ and $\varphi_b = \arg b$, then

$$\log_b z = \log_b(r_a e^{i\varphi_a}) = \frac{\ln r_a + i\varphi_a}{\ln b} = \frac{\ln r_a + i\varphi_a}{\ln r_b + i\varphi_b} = \frac{\ln z}{\ln b}$$

This uses the change of base formula, which says

$$\log_b x = \frac{\log_k x}{\log_k b}$$

By the definition of a logarithm,

$$x = b^a \iff \log_b x = a$$

and using the base k logarithm,

$$\log_k x = \log_k b^a = a \log_k b \iff \frac{\log_k x}{\log_k b} = a$$

Since the natural logarithm is defined by the standard C library, and the desired base is b , we use $k = e$ and $b = b$.

Parameters

in	z	The ComplexNumber of which the logarithm base b is to be found
in	b	The real value that is the base of the logarithm

Returns

A [ComplexNumber](#) that is the base b logarithm of a [ComplexNumber](#)

Definition at line 168 of file ComplexMath.cpp.

5.1.4.45 `template<class T> ComplexNumber< T > logb (const T & x, const ComplexNumber< T > & b)` [related]

Logarithm base b of a real value.

Computes the complex base b logarithm of a real value. This is useful for when $x < 0$.

If $z = r_a e^{i\varphi_a}$ and $b = r_b e^{i\varphi_b}$, where $r_a = |z|$ and $\varphi_a = \arg z$ and $r_b = |b|$ and $\varphi_b = \arg b$, then

$$\log_b z = \log_b(r_a e^{i\varphi_a}) = \frac{\ln r_a + i\varphi_a}{\ln b} = \frac{\ln r_a + i\varphi_a}{\ln r_b + i\varphi_b} = \frac{\ln z}{\ln b}$$

This uses the change of base formula, which says

$$\log_b x = \frac{\log_k x}{\log_k b}$$

By the definition of a logarithm,

$$x = b^a \iff \log_b x = a$$

and using the base k logarithm,

$$\log_k x = \log_k b^a = a \log_k b \iff \frac{\log_k x}{\log_k b} = a$$

Since the natural logarithm is defined by the standard C library, and the desired base is b , we use $k = e$ and $b = b$.

Parameters

in	x	The real value of which the logarithm base b is to be found
in	b	The ComplexNumber that is the base of the logarithm

Returns

A [ComplexNumber](#) that is the base b logarithm of a real value

Definition at line 174 of file ComplexMath.cpp.

5.1.4.46 `template<class T> ComplexNumber< T > logb (const T & x, const T & b)`
[\[related\]](#)

Logarithm base b of a real value.

Computes the real base b logarithm of a real value. This is useful for when $x < 0$ and/or $b < 0$.

If $x = r_a e^{i\varphi_a}$ and $b = r_b e^{i\varphi_b}$, where $r_a = |x|$ and $\varphi_a = \arg x$ and $r_b = |b|$ and $\varphi_b = \arg b$, then

$$\log_b z = \log_b(r_a e^{i\varphi_a}) = \frac{\ln r_a + i\varphi_a}{\ln b} = \frac{\ln r_a + i\varphi_a}{\ln r_b + i\varphi_b} = \frac{\ln z}{\ln b}$$

This uses the change of base formula, which says

$$\log_b x = \frac{\log_k x}{\log_k b}$$

By the definition of a logarithm,

$$x = b^a \iff \log_b x = a$$

and using the base k logarithm,

$$\log_k x = \log_k b^a = a \log_k b \iff \frac{\log_k x}{\log_k b} = a$$

Since the natural logarithm is defined by the standard C library, and the desired base is b , we use $k = e$ and $b = b$.

Parameters

<code>x</code>	The real value of which the logarithm base b is to be found
<code>b</code>	The real value that is the base of the logarithm

Returns

A [ComplexNumber](#) that is the base b logarithm of a real value

Definition at line 180 of file ComplexMath.cpp.

5.1.4.47 `template<class T> ComplexNumber< T > nthRoot (const ComplexNumber< T > & z, const int n)` [related]

n th root of a [ComplexNumber](#)

Computes the principle n th root of a [ComplexNumber](#).

If $z = r e^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$, then

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{i\frac{\varphi}{n}} = (a^2 + b^2)^{\frac{1}{2n}} \left(\cos\left(\frac{\varphi}{n}\right) + i \sin\left(\frac{\varphi}{n}\right) \right)$$

This is a result of de Moivre's formula,

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$

which can be proven by Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

and the rules of exponentiation,

$$(e^{ix})^n = e^{inx}$$

So,

$$e^{i(nx)} = (\cos x + i \sin x)^n = \cos nx + i \sin nx$$

Parameters

in	z	The ComplexNumber of which to compute the nth root
in	n	The root to which the ComplexNumber is evaluated

Returns

A [ComplexNumber](#) that is the nth root of a [ComplexNumber](#)

Definition at line 253 of file ComplexMath.cpp.

5.1.4.48 `template<class T> std::vector< ComplexNumber< T > > nthRoots (const ComplexNumber< T > & z, const int n)` [related]

nth roots of a [ComplexNumber](#)

Computes the nth roots of a [ComplexNumber](#).

If $z = re^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$, then

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{i\frac{\varphi}{n}} = (a^2 + b^2)^{\frac{1}{2n}} \left(\cos \left(\frac{\varphi + 2k\pi}{n} \right) + i \sin \left(\frac{\varphi + 2k\pi}{n} \right) \right)$$

Where $\{k \in \mathbb{Z} | 0 \leq k < n\}$

This is a result of de Moivre's formula,

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$

which can be proven by Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

and the rules of exponentiation,

$$(e^{ix})^n = e^{inx}$$

So,

$$e^{i(n x)} = (\cos x + i \sin x)^n = \cos nx + i \sin nx$$

Parameters

in	z	The ComplexNumber of which to compute the nth roots
in	n	The root to which the ComplexNumber is evaluated

Returns

A vector containing the ComplexNumbers that are the n th roots of a [ComplexNumber](#)

Definition at line 267 of file ComplexMath.cpp.

5.1.4.49 `template<class T> void nthRoots (const ComplexNumber< T > & z, std::vector< ComplexNumber< T >> roots, const int n)` [related]

n th roots of a [ComplexNumber](#)

Computes the n th roots of a [ComplexNumber](#).

If $z = re^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$, then

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{i\frac{\varphi}{n}} = (a^2 + b^2)^{\frac{1}{2n}} \left(\cos \left(\frac{\varphi + 2k\pi}{n} \right) + i \sin \left(\frac{\varphi + 2k\pi}{n} \right) \right)$$

Where $\{k \in \mathbb{Z} | 0 \leq k < n\}$

This is a result of de Moivre's formula,

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$

which can be proven by Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

and the rules of exponentiation,

$$(e^{ix})^n = e^{inx}$$

So,

$$e^{i(nx)} = (\cos x + i \sin x)^n = \cos nx + i \sin nx$$

Parameters

in	z	The ComplexNumber of which to compute the n th roots
in	$roots$	The vector containing the ComplexNumbers that are the n th roots of a ComplexNumber
in	n	The root to which the ComplexNumber is evaluated

5.1.4.50 `template<class T, class U > bool operator!=(const ComplexNumber< T > & lhs, const ComplexNumber< U > & rhs)` [related]

Compares a [ComplexNumber](#) to another [ComplexNumber](#).

Determines if the [ComplexNumber](#) on the left side of the operand is not equal to the [ComplexNumber](#) on the right side of the operand

Parameters

<code>in</code>	<code>lhs</code>	The ComplexNumber on the left side of the operand to which the ComplexNumber on the right will be compared
<code>in</code>	<code>rhs</code>	The ComplexNumber on the right side of the operand, which will be compared to the the ComplexNumber on the left side of the operand

Returns

A boolean indicating whether the [ComplexNumber](#) on the left side of the operand is not equal to the [ComplexNumber](#) on the right side of the operand

Definition at line 355 of file ComplexNumber.cpp.

5.1.4.51 `template<class T, class U > bool operator!=(const ComplexNumber< T > & lhs, const U & rhs)` [related]

Compares a [ComplexNumber](#) to a real value.

Determines if the [ComplexNumber](#) on the left side of the operand is not equal to the real value on the right side of the operand

Parameters

<code>in</code>	<code>lhs</code>	The ComplexNumber on the left side of the operand to which the real value on the right will be compared
<code>in</code>	<code>rhs</code>	The real value on the right side of the operand, which will be compared to the the ComplexNumber on the left side of the operand

Returns

A boolean indicating whether the [ComplexNumber](#) on the left side of the operand is not equal to the real value on the right side of the operand

Definition at line 361 of file ComplexNumber.cpp.

5.1.4.52 `template<class T, class U > bool operator!=(const T & lhs, const ComplexNumber< U > & rhs)` [related]

Compares a real value to a [ComplexNumber](#).

Determines if the real value on the left side of the operand is not equal to the [ComplexNumber](#) on the right side of the operand

Parameters

in	<i>lhs</i>	The real value on the left side of the operand to which the ComplexNumber on the right will be compared
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will be compared to the the real value on the left side of the operand

Returns

A boolean indicating whether the real value on the left side of the operand is not equal to the [ComplexNumber](#) on the right side of the operand

Definition at line 366 of file `ComplexNumber.cpp`.

5.1.4.53 `template<class T > ComplexNumber< T > operator*(const ComplexNumber< T > & lhs, const ComplexNumber< T > & rhs)` [related]

Multiplies a [ComplexNumber](#) by another [ComplexNumber](#).

Multiplies a [ComplexNumber](#) on the left side of the operand by a [ComplexNumber](#) on the left side of the operand, returning the resultant product as a third [ComplexNumber](#).

$$z_3 = z_1 z_2 = (a_1 + b_1 i)(a_2 + b_2 i) = a_1 a_2 + (b_1 a_2) i + (a_1 b_2) i + (b_1 b_2) i^2 = (a_1 a_2 - b_1 b_2) + (b_1 a_2 + a_1 b_2) i$$

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand, by which the ComplexNumber on the right side of the operand will be multiplied
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, by which the ComplexNumber on the left side of the operand will be multiplied

Returns

A [ComplexNumber](#) consisting of the product of the two [ComplexNumbers](#)

Definition at line 280 of file `ComplexNumber.cpp`.

5.1.4.54 `template<class T > ComplexNumber< T > operator* (const
ComplexNumber< T > & lhs, const T & rhs)` [related]

Multiplies a [ComplexNumber](#) by a real value.

Multiplies the real and imaginary values of the [ComplexNumber](#) on the left side of the operand by a real number on the right side of the operand, returning the resultant product as a third [ComplexNumber](#).

$$z_2 = z_1x = (a_1 + b_1i)x = (a_1x) + (b_1x)i$$

Parameters

<code>in</code>	<code>lhs</code>	The ComplexNumber on the left side of the operand, by which the real value on the right side of the operand will be multiplied
<code>in</code>	<code>rhs</code>	The real value on the right side of the operand, by which the ComplexNumber on the left side of the operand will be multiplied

Returns

A [ComplexNumber](#) consisting of the product of the [ComplexNumber](#) and the real value

Definition at line 289 of file `ComplexNumber.cpp`.

5.1.4.55 `template<class T > ComplexNumber< T > operator* (const T & lhs, const
ComplexNumber< T > & rhs)` [related]

Multiplies a real value by a [ComplexNumber](#).

Multiplies a real value on the left side of the operand by the real and imaginary values of a [ComplexNumber](#) on the right side of the operand, returning the resultant product as a third [ComplexNumber](#).

$$z_3 = xz_1 = x(a_1 + b_1i) = (xa_1) + (xb_1)i$$

Parameters

in	<i>lhs</i>	The real value on the left side of the operand, by which the ComplexNumber on the right side of the operand will be multiplied
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, by which the ComplexNumber on the left side of the operand will be multiplied

Returns

A [ComplexNumber](#) consisting of the product of the [ComplexNumber](#) and the real value

Definition at line 294 of file ComplexNumber.cpp.

5.1.4.56 `template<class T > ComplexNumber< T > operator+ (const ComplexNumber< T > & lhs, const ComplexNumber< T > & rhs)`
[related]

Adds a [ComplexNumber](#) to another [ComplexNumber](#).

Adds the real and imaginary values of a [ComplexNumber](#) on the right and left side of the operand, returning the resultant sum as a third [ComplexNumber](#).

$$z_3 = z_1 + z_2 = (a_1 + b_1i) + (a_2 + b_2i) = (a_1 + a_2) + (b_1 + b_2)i$$

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand, to which the ComplexNumber on the right side of the operand will be added
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will be added to the ComplexNumber on the left side of the operand

Returns

A [ComplexNumber](#) consisting of the sum of the two [ComplexNumbers](#)

Definition at line 246 of file ComplexNumber.cpp.

5.1.4.57 `template<class T > ComplexNumber< T > operator+ (const ComplexNumber< T > & lhs, const T & rhs)` [related]

Adds a real value to a [ComplexNumber](#).

Adds the real value of a [ComplexNumber](#) on the left side of the operand and a real number on the right side of the operand, returning the resultant sum as a third [ComplexNumber](#).

$$z_2 = z_1 + x = (a_1 + b_1 i) + x = (a_1 + x) + b_1 i$$

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand, to which the real value on the right side of the operand will be added
in	<i>rhs</i>	The real value on the right side of the operand, which will be added to the ComplexNumber on the left side of the operand

Returns

A [ComplexNumber](#) consisting of the sum of the [ComplexNumber](#) and the real value

Definition at line 253 of file ComplexNumber.cpp.

5.1.4.58 `template<class T > ComplexNumber< T > operator+ (const T & lhs, const ComplexNumber< T > & rhs)` [related]

Adds a [ComplexNumber](#) to a real value.

Adds a real value on the left side of the operand the real value of a [ComplexNumber](#) on the right side of the operand, returning the resultant sum as a third [ComplexNumber](#).

$$z_2 = x + z_1 = x + (a_1 + b_1 i) = (x + a_1) + b_1 i$$

Parameters

in	<i>lhs</i>	The real value on the right side of the operand, to which the ComplexNumber on the right side of the operand will be added
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will be added to the real value on the left side of the operand

Returns

A [ComplexNumber](#) consisting of the sum of the [ComplexNumber](#) and the real value

Definition at line 258 of file ComplexNumber.cpp.

5.1.4.59 `template<class T > ComplexNumber< T > operator- (const ComplexNumber< T > & lhs, const ComplexNumber< T > & rhs)` [\[related\]](#)

Subtracts a [ComplexNumber](#) from another [ComplexNumber](#).

Subtracts the real and imaginary values of a [ComplexNumber](#) on the right side of the operand from those of the [ComplexNumber](#) on the left side of the operand, returning the resultant difference as a third [ComplexNumber](#).

$$z_3 = z_1 - z_2 = (a_1 + b_1i) - (a_2 + b_2i) = (a_1 - a_2) + (b_1 - b_2)i$$

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand, from which the ComplexNumber on the right side of the operand will be subtracted
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will be subtracted from the ComplexNumber on the left side of the operand

Returns

A [ComplexNumber](#) consisting of the difference of the two [ComplexNumbers](#)

Definition at line 263 of file ComplexNumber.cpp.

5.1.4.60 `template<class T > ComplexNumber< T > operator- (const ComplexNumber< T > & lhs, const T & rhs)` [\[related\]](#)

Subtracts a real value from a [ComplexNumber](#).

Subtracts a real value on the right side of the operand from the real value of the [ComplexNumber](#) on the left side of the operand, returning the resultant difference as a third [ComplexNumber](#).

$$z_2 = z_1 - x = (a_1 + b_1i) - x = (a_1 - x) + b_1i$$

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand, from which the real value on the right side of the operand will be subtracted
in	<i>rhs</i>	The real value on the right side of the operand, which will be subtracted from the ComplexNumber on the left side of the operand

Returns

A [ComplexNumber](#) consisting of the difference of the [ComplexNumber](#) and the real value

Definition at line 270 of file ComplexNumber.cpp.

5.1.4.61 `template<class T > ComplexNumber< T > operator- (const T & lhs, const ComplexNumber< T > & rhs)` [[related](#)]

Subtracts a [ComplexNumber](#) from a real value.

Subtracts the real value of the [ComplexNumber](#) on the right side of the operand from a real value on the left side of the operand, returning the resultant difference as a third [ComplexNumber](#).

$$z_2 = x - z_1 = x - (a_1 + b_1 i) = (x - a_1) + b_1 i$$

Parameters

in	<i>lhs</i>	The real value on the left side of the operand, from which the ComplexNumber on the right side of the operand will be subtracted
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will be subtracted from the real value on the left side of the operand

Returns

A [ComplexNumber](#) consisting of the difference of the [ComplexNumber](#) and the real value

Definition at line 275 of file ComplexNumber.cpp.

5.1.4.62 `template<class T > ComplexNumber< T > operator/ (const ComplexNumber< T > & lhs, const ComplexNumber< T > & rhs)` [\[related\]](#)

Divides a [ComplexNumber](#) by another [ComplexNumber](#).

Divides the [ComplexNumber](#) on the left side of the operand by the [ComplexNumber](#) on the right side of the operand, using the complex conjugate of the denominator, returning the resultant quotient as a third [ComplexNumber](#).

$$z_3 = \frac{z_1}{z_2} = \frac{a_1 + b_1i}{a_2 + b_2i} = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} + \frac{b_1a_2 - a_1b_2}{a_2^2 + b_2^2}i$$

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand, by which the ComplexNumber on the right side of the operand will be divided
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will divide the ComplexNumber on the left side of the operand

Exceptions

DivisionByZeroException	if attempting to divide a ComplexNumber lhs by a ComplexNumber rhs that is equal to zero, a DivisionByZeroException is thrown
---	---

Returns

A [ComplexNumber](#) consisting of the quotient of the two [ComplexNumbers](#)

Definition at line 299 of file `ComplexNumber.cpp`.

5.1.4.63 `template<class T > ComplexNumber< T > operator/ (const ComplexNumber< T > & lhs, const T & rhs)` [\[related\]](#)

Divides a [ComplexNumber](#) by a real value.

Divides the real and imaginary values of a [ComplexNumber](#) on the left side of the operand by a real value on the right side of the operand, returning the resultant quotient as a third [ComplexNumber](#).

$$z_2 = \frac{z_1}{x} = \frac{a_1 + b_1i}{x} = \frac{a_1}{x} + \frac{b_1}{x}i$$

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand, by which the real value on the right side of the operand will be divided
in	<i>rhs</i>	The real value on the right side of the operand, which will divide the ComplexNumber on the left side of the operand

Exceptions

DivisionByZero-Exception	if attempting to divide a ComplexNumber lhs by a value rhs that is equal to zero, a DivisionByZeroException is thrown
--	---

Returns

A [ComplexNumber](#) consisting of the quotient of the [ComplexNumber](#) and the real value

Definition at line 316 of file ComplexNumber.cpp.

5.1.4.64 `template<class T > ComplexNumber< T > operator/ (const T & lhs, const ComplexNumber< T > & rhs)` [related]

Divides a real value by a [ComplexNumber](#).

Divides a real value on the left side of the operand by a [ComplexNumber](#) on the right side of the operand, returning the resultant quotient as a third [ComplexNumber](#).

$$z_2 = \frac{x}{z_1} = \frac{x}{a_1 + b_1 i} = \frac{a_1 x}{a_1^2 + b_1^2} - \frac{b_1 x}{a_1^2 + b_1^2} i$$

Parameters

in	<i>lhs</i>	The real value on the left side of the operand, by which the ComplexNumber on the right side of the operand will be divided
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will divide the real value on the left side of the operand

Exceptions

DivisionByZero-Exception	if attempting to divide a value lhs by a ComplexNumber rhs that is equal to zero, a DivisionByZeroException is thrown
--	---

Returns

A [ComplexNumber](#) consisting of the quotient of the real value and the [ComplexNumber](#)

Definition at line 325 of file ComplexNumber.cpp.

```
5.1.4.65 template<class U > std::ostream& operator<< ( std::ostream & out, const
          ComplexNumber< U > & z ) [friend]
```

Inserts a [ComplexNumber](#) into the ostream.

Sends a representation of this [ComplexNumber](#) into the ostream

Parameters

in	<i>out</i>	The ostream reference into which the ComplexNumber will be inserted
in	<i>z</i>	The ComplexNumber from which the data will be obtained

Returns

A reference to the modified ostream

Definition at line 371 of file ComplexNumber.cpp.

```
5.1.4.66 template<class T , class U > bool operator== ( const ComplexNumber< T > &
          lhs, const ComplexNumber< U > & rhs ) [related]
```

Compares a [ComplexNumber](#) to another [ComplexNumber](#).

Determines if the [ComplexNumber](#) on the left side of the operand is equal to the [ComplexNumber](#) on the right side of the operand

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand to which the ComplexNumber on the right will be compared
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will be compared to the the ComplexNumber on the left side of the operand

Returns

A boolean indicating whether the [ComplexNumber](#) on the left side of the operand is equal to the [ComplexNumber](#) on the right side of the operand

Definition at line 339 of file ComplexNumber.cpp.

```
5.1.4.67 template<class T , class U > bool operator==( const ComplexNumber< T > &
           lhs, const U & rhs ) [related]
```

Compares a [ComplexNumber](#) to a real value.

Determines if the [ComplexNumber](#) on the left side of the operand is equal to the real value on the right side of the operand

Parameters

in	<i>lhs</i>	The ComplexNumber on the left side of the operand to which the real value on the right will be compared
in	<i>rhs</i>	The real value on the right side of the operand, which will be compared to the the ComplexNumber on the left side of the operand

Returns

A boolean indicating whether the [ComplexNumber](#) on the left side of the operand is equal to the real value on the right side of the operand

Definition at line 345 of file ComplexNumber.cpp.

```
5.1.4.68 template<class T , class U > bool operator==( const T & lhs, const
           ComplexNumber< U > & rhs ) [related]
```

Compares a real value to a [ComplexNumber](#).

Determines if the real value on the left side of the operand is equal to the [ComplexNumber](#) on the right side of the operand

Parameters

in	<i>lhs</i>	The real value on the left side of the operand to which the ComplexNumber on the right will be compared
in	<i>rhs</i>	The ComplexNumber on the right side of the operand, which will be compared to the the real value on the left side of the operand

Returns

A boolean indicating whether the real value on the left side of the operand is equal to the [ComplexNumber](#) on the right side of the operand

Definition at line 350 of file ComplexNumber.cpp.

5.1.4.69 `template<class U > std::istream& operator>> (std::istream & in, ComplexNumber< U > & z) [friend]`

Extracts a [ComplexNumber](#) from the istream.

Initializes a [ComplexNumber](#) from values extracted from the istream

Parameters

<code>in</code>	<code>in</code>	The istream reference from which the ComplexNumber will be extracted
<code>in</code>	<code>z</code>	The ComplexNumber into which the data will be inserted

Returns

A reference to the modified istream

Definition at line 383 of file ComplexNumber.cpp.

5.1.4.70 `template<class T > ComplexNumber< T > pow (const ComplexNumber< T > & a, const ComplexNumber< T > & b) [related]`

Exponentiation of ComplexNumbers.

Computes the exponentiation of a [ComplexNumber](#) and another [ComplexNumber](#).

$$a^b = e^{\ln a^b} = e^{b \ln a} = e^{b \ln |a| + i \arg a}$$

Parameters

<code>in</code>	<code>a</code>	The ComplexNumber that is the base of the exponentiation
<code>in</code>	<code>b</code>	The ComplexNumber that is the exponent of the exponentiation

Returns

A [ComplexNumber](#) that is the exponentiation of two ComplexNumbers

Definition at line 186 of file ComplexMath.cpp.

5.1.4.71 `template<class T> ComplexNumber< T > pow (const ComplexNumber< T > & a, const T & b)` [related]

Exponentiation of a [ComplexNumber](#) and real value.

Computes the exponentiation of a [ComplexNumber](#) and a real value.

$$a^b = e^{\ln a^b} = e^{b \ln a} = e^{b \ln |a| + i \arg a}$$

Parameters

in	<i>a</i>	The ComplexNumber that is the base of the exponentiation
in	<i>b</i>	The real value that is the exponent of the exponentiation

Returns

A [ComplexNumber](#) that is the exponentiation of a [ComplexNumber](#) and a real value

Definition at line 199 of file ComplexMath.cpp.

5.1.4.72 `template<class T> ComplexNumber< T > pow (const T & a, const ComplexNumber< T > & b)` [related]

Exponentiation of a real value and a [ComplexNumber](#).

Computes the exponentiation of a real value and a [ComplexNumber](#).

$$a^b = e^{\ln a^b} = e^{b \ln a} = e^{b \ln |a| + i \arg a}$$

Parameters

in	<i>a</i>	The real value that is the base of the exponentiation
in	<i>b</i>	The ComplexNumber that is the exponent of the exponentiation

Returns

A [ComplexNumber](#) that is the exponentiation of a real value and a [ComplexNumber](#)

Definition at line 208 of file ComplexMath.cpp.

5.1.4.73 `template<class T> ComplexNumber< T > sec (const ComplexNumber< T > & z)` [related]

Secant of a [ComplexNumber](#).

Computes the complex secant of a [ComplexNumber](#).

$$\sec z = \frac{1}{\cos z}$$

Parameters

in	z	The ComplexNumber of which to compute the secant
----	---	--

Returns

A [ComplexNumber](#) that is the secant of a [ComplexNumber](#)

Definition at line 324 of file ComplexMath.cpp.

5.1.4.74 `template<class T> ComplexNumber< T > sech (const ComplexNumber< T > & z)` [related]

Hyperbolic secant of a [ComplexNumber](#).

Computes the complex hyperbolic secant of a [ComplexNumber](#).

$$\operatorname{sech} z = \frac{1}{\operatorname{cosh} z}$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic secant
----	---	---

Returns

A [ComplexNumber](#) that is the hyperbolic secant of a [ComplexNumber](#)

Definition at line 469 of file ComplexMath.cpp.

5.1.4.75 `template<class T> ComplexNumber< T > sin (const ComplexNumber< T > & z)` [related]

Sine of a [ComplexNumber](#).

Computes the complex sine of a [ComplexNumber](#).

$$\sin z = \sin(a + bi) = \sin a \cos(bi) + \cos a \sin(bi) = \sin a \cosh b + i \cos a \sinh b$$

This is a result of the addition or subtraction theorems or formulae, which can be proven by Euler's formula, $e^{ix} = \cos x + i \sin x$, and by noting the symmetry of the sine and cosine functions as follows,

$$\begin{aligned} \cos(\alpha \pm \beta) + i \sin(\alpha \pm \beta) &= e^{i(\alpha \pm \beta)} \\ &= e^{i\alpha} e^{\pm i\beta} \\ &= (\cos \alpha + i \sin \alpha)(\cos \beta \pm i \sin \beta) \\ &= (\cos \alpha \cos \beta \mp \sin \alpha \sin \beta) + i(\sin \alpha \cos \beta \pm \cos \alpha \sin \beta) \end{aligned}$$

Thus,

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \end{aligned}$$

Since, by the exponential definitions of the trigonometric functions derived from Euler's formula and the definitions of the hyperbolic functions,

$$\begin{aligned} \sin x &= \frac{e^{ix} - e^{-ix}}{2i} \\ \cos x &= \frac{e^{ix} + e^{-ix}}{2} \\ \sinh x &= \frac{e^x - e^{-x}}{2} \\ \cosh x &= \frac{e^x + e^{-x}}{2} \end{aligned}$$

Then,

$$\begin{aligned}\sin(xi) &= \frac{e^{i^2x} - e^{-i^2x}}{2i} = \frac{e^{-x} - e^x}{2i} = \frac{e^x - e^{-x}}{2}i = i \sinh x \\ \cos(xi) &= \frac{e^{i^2x} + e^{-i^2x}}{2} = \frac{e^{-x} + e^x}{2} = \cosh x\end{aligned}$$

Parameters

in	z	The ComplexNumber of which to compute the sine
----	---	--

Returns

A [ComplexNumber](#) that is the sine of a [ComplexNumber](#)

Definition at line 293 of file ComplexMath.cpp.

5.1.4.76 `template<class T> ComplexNumber< T > sinh (const ComplexNumber< T > & z)` [related]

Hyperbolic sine of a [ComplexNumber](#).

Computes the complex hyperbolic sine of a [ComplexNumber](#).

$$\sinh z = \sinh(a + bi) = \sinh a \cosh(bi) + \cosh a \sinh(bi) = \sinh a \cos b + i \cosh a \sin b$$

This is a result of the addition or subtraction theorems or formulae,

$$\begin{aligned}\sinh(x \pm y) &= \sinh x \cosh y \pm \cosh x \sinh y \\ \cosh(x \pm y) &= \cosh x \cosh y \pm \sinh x \sinh y\end{aligned}$$

which can be proven as follows, also noting the symmetry of the hyperbolic sine and hyperbolic cosine functions,

$$\begin{aligned}
\sinh(x+y) &= \frac{e^{x+y} - e^{-x-y}}{2} \\
&= \frac{e^x e^y - e^x e^{-y} + e^x e^{-y} - e^{-x} e^{-y}}{2} \\
&= e^x \left(\frac{e^y - e^{-y}}{2} \right) + e^{-y} \left(\frac{e^x - e^{-x}}{2} \right) \\
&= (\cosh x + \sinh x) \sinh y + (\cosh y - \sinh y) \sinh x \\
&= \cosh x \sinh y + \sinh x \sinh y + \sinh x \cosh y - \sinh x \sinh y \\
&= \sinh x \cosh y + \cosh x \sinh y
\end{aligned}$$

$$\begin{aligned}
\cosh(x+y) &= \frac{e^{x+y} + e^{-x-y}}{2} \\
&= \frac{e^x e^y - e^x e^{-y} + e^x e^{-y} + e^{-x} e^{-y}}{2} \\
&= e^x \left(\frac{e^y - e^{-y}}{2} \right) + e^{-y} \left(\frac{e^x + e^{-x}}{2} \right) \\
&= (\cosh x + \sinh x) \sinh y + (\cosh y - \sinh y) \cosh x \\
&= \cosh x \sinh y + \sinh x \sinh y + \cosh x \cosh y - \cosh x \sinh y \\
&= \cosh x \cosh y + \sinh x \sinh y
\end{aligned}$$

Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

Since, by the exponential definitions of the trigonometric functions derived from Euler's formula and the definitions of the hyperbolic functions,

$$\begin{aligned}
\sin x &= \frac{e^{ix} - e^{-ix}}{2i} \\
\cos x &= \frac{e^{ix} + e^{-ix}}{2} \\
\sinh x &= \frac{e^x - e^{-x}}{2} \\
\cosh x &= \frac{e^x + e^{-x}}{2}
\end{aligned}$$

Then,

$$\sinh(xi) = \frac{e^{xi} - e^{-xi}}{2} = i \sin x$$

$$\cosh(xi) = \frac{e^{xi} + e^{-xi}}{2} = \cos x$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic sine
----	---	---

Returns

A [ComplexNumber](#) that is the hyperbolic sine of a [ComplexNumber](#)

Definition at line 438 of file ComplexMath.cpp.

5.1.4.77 `template<class T> ComplexNumber< T > sqrt (const ComplexNumber< T > & z)` [related]

Square root of a [ComplexNumber](#).

Computes the principle square root of a [ComplexNumber](#).

If $z = re^{i\varphi}$, where $r = |z|$ and $\varphi = \arg z$, then

$$\sqrt{z} = \sqrt{r}e^{i\frac{\varphi}{2}} = (a^2 + b^2)^{\frac{1}{4}} \left(\cos\left(\frac{\varphi}{2}\right) + i \sin\left(\frac{\varphi}{2}\right) \right)$$

This is a result of de Moivre's formula,

$$(\cos x + i \sin x)^n = \cos nx + i \sin nx$$

which can be proven by Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

and the rules of exponentiation,

$$(e^{ix})^n = e^{inx}$$

So,

$$e^{i(nx)} = (\cos x + i \sin x)^n = \cos nx + i \sin nx$$

Parameters

in	z	The ComplexNumber of which to compute the square roots
----	---	--

Returns

A [ComplexNumber](#) that is the square root of a [ComplexNumber](#)

Definition at line 221 of file ComplexMath.cpp.

5.1.4.78 `template<class T> ComplexNumber< T > sqrt (const T & x)` [related]

Square root of a real value.

Computes the principle square root of a real value.

If $x < 0$, the solutions are both real numbers and can be computed as usual.

If $x < 0$, since $i^2 = -1$, we can factor i out of the radical and compute the square root of x as usual, resulting in imaginary solutions.

Parameters

in	x	The real value of which to compute the square root
----	---	--

Returns

A [ComplexNumber](#) that is the square root of a real value

Definition at line 232 of file ComplexMath.cpp.

5.1.4.79 `template<class T> ComplexNumber< T > tan (const ComplexNumber< T > & z)` [related]

Tangent of a [ComplexNumber](#).

Computes the complex tangent of a [ComplexNumber](#).

$$\tan z = \tan(a + bi) = \frac{\sin 2a}{\cos 2a + \cosh 2b} + \frac{\sinh 2b}{\cos 2a + \cosh 2a} i$$

This is a result of the exponential definitions for the trigonometric functions derived from Euler's formula and the definitions of the hyperbolic functions,

$$\begin{aligned}\sin x &= \frac{e^{ix} - e^{-ix}}{2i} \\ \cos x &= \frac{e^{ix} + e^{-ix}}{2} \\ \tan x &= \frac{\sin x}{\cos x} = -i \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \\ \sinh x &= \frac{e^x - e^{-x}}{2} \\ \cosh x &= \frac{e^x + e^{-x}}{2}\end{aligned}$$

Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

and the double angle formulae for sine and cosine, which can be proven by either the multiple-angle formulae or the angle sum and difference identities,

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta\end{aligned}$$

So,

$$\begin{aligned}\tan z &= \tan(a + bi) \\ &= -i \frac{e^{i(a+bi)} - e^{-i(a+bi)}}{e^{i(a+bi)} + e^{-i(a+bi)}} \\ &= -i \frac{e^{ai-b} - e^{b-ai}}{e^{ai-b} + e^{b-ai}} \\ &= -i \frac{e^{ai}e^{-b} - e^b e^{-ai}}{e^{ai}e^{-b} + e^b e^{-ai}} \\ &= -i \frac{e^{-b}(\cos a + i \sin a) - e^b(\cos a - i \sin a)}{e^{-b}(\cos a + i \sin a) + e^b(\cos a - i \sin a)} \\ &= \frac{(e^{-b} \sin a + e^b \sin a) + (e^b \cos a + e^{-b} \cos a)i}{(e^{-b} \cos a + e^b \cos a) + (e^{-b} \sin a - e^b \sin a)i}\end{aligned}$$

Multiplying the numerator and denominator by the complex conjugate of the denominator,

$$\begin{aligned}
& \frac{(e^{-b} \sin a + e^b \sin a)(e^{-b} \cos a + e^b \cos a) + (e^b \cos a - e^{-b} \cos a)(e^{-b} \sin a - e^b \sin a)}{(e^{-b} \cos a + e^b \cos a)^2 + (e^{-b} \sin a - e^b \sin a)^2} + \\
& \frac{(e^b \cos a - e^{-b} \cos a)(e^{-b} \cos a + e^b \cos a) - (e^{-b} \sin a + e^b \sin a)(e^{-b} \sin a - e^b \sin a)}{(e^{-b} \cos a + e^b \cos a)^2 + (e^{-b} \sin a - e^b \sin a)^2} i = \\
& \frac{(4 \sin a \cos a) + (e^{2b} \cos^2 a - e^{-2b} \cos^2 a + e^{2b} \sin^2 a - e^{-2b} \sin^2 a) i}{e^{2b} \cos^2 a + e^{-2b} \cos^2 a + 2 \cos^2 a + e^{2b} \sin^2 a + e^{-2b} \sin^2 a - 2 \sin^2 a} = \\
& \frac{(4 \sin a \cos a) + (e^{2b} - e^{-2b})(\cos^2 a + \sin^2 a) i}{(e^{2b} + e^{-2b})(\cos^2 a + \sin^2 a) + 2(\cos^2 a - \sin^2 a)} = \\
& \frac{2 \left(2 \sin a \cos a + \frac{e^{2b} - e^{-2b}}{2} i \right)}{2 \left(\frac{e^{2b} + e^{-2b}}{2} + \cos^2 a - \sin^2 a \right)} = \\
& \frac{\sin 2a}{\cos 2a + \cosh 2b} + \frac{\sinh 2b}{\cos 2a + \cosh 2b} i
\end{aligned}$$

Parameters

in	z	The ComplexNumber of which to compute the tangent
----	---	---

Returns

A [ComplexNumber](#) that is the tangent of a [ComplexNumber](#)

Definition at line 307 of file ComplexMath.cpp.

5.1.4.80 `template<class T> ComplexNumber< T > tanh (const ComplexNumber< T > & z)` [related]

Hyperbolic tangent of a [ComplexNumber](#).

Computes the complex hyperbolic tangent of a [ComplexNumber](#).

$$\tanh z = \tanh(a + bi) = \frac{\sinh 2a}{\cosh 2a + \cos 2b} + \frac{\sin 2b}{\cosh 2a + \cos 2b} i$$

This is a result of the definitions for the hyperbolic functions,

$$\begin{aligned}\sinh x &= \frac{e^x - e^{-x}}{2} \\ \cosh x &= \frac{e^x + e^{-x}}{2} \\ \tanh x &= \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}\end{aligned}$$

Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

and the double angle formulae for sine and cosine, which can be proven by either the multiple-angle formulae or the angle sum and difference identities,

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta\end{aligned}$$

So,

$$\begin{aligned}\tanh z &= \tanh(a + bi) \\ &= \frac{e^{a+bi} - e^{-a-bi}}{e^{a+bi} + e^{-a-bi}} \\ &= \frac{e^a e^{bi} - e^{-a} e^{-bi}}{e^a e^{bi} + e^{-a} e^{-bi}} \\ &= \frac{e^a(\cos b + i \sin b) - e^{-a}(\cos b - i \sin b)}{e^a(\cos b + i \sin b) + e^{-a}(\cos b - i \sin b)} \\ &= \frac{(e^a \cos b - e^{-a} \cos b) + (e^a \sin b + e^{-a} \sin b)i}{(e^a \cos b + e^{-a} \cos b) + (e^a \sin b - e^{-a} \sin b)i}\end{aligned}$$

Multiplying the numerator and denominator by the complex conjugate of the denominator,

$$\begin{aligned}
& \frac{(e^a \cos b - e^{-a} \cos b)(e^a \cos b + e^{-a} \cos b) + (e^a \sin b + e^{-a} \sin b)(e^a \sin b - e^{-a} \sin b)}{(e^a \cos b + e^{-a} \cos b)^2 + (e^a \sin b - e^{-a} \sin b)^2} + \\
& \frac{(e^a \sin b + e^{-a} \sin b)(e^a \cos b + e^{-a} \cos b) - (e^a \cos b - e^{-a} \cos b)(e^a \sin b - e^{-a} \sin b)}{(e^a \cos b + e^{-a} \cos b)^2 + (e^a \sin b - e^{-a} \sin b)^2} i = \\
& \frac{(e^{2a} \cos^2 b - e^{-2a} \cos^2 b + e^{2a} \sin^2 b - e^{-2a} \sin^2 b) + (4 \sin b \cos b) i}{e^{2a} \cos^2 b + e^{-2a} \cos^2 b + 2 \cos^2 b + e^{2a} \sin^2 b + e^{-2a} \sin^2 b - 2 \sin^2 b} = \\
& \frac{(e^{2a} - e^{-2a})(\cos^2 b + \sin^2 b) + (4 \sin b \cos b) i}{(e^{2a} + e^{-2a})(\cos^2 b + \sin^2 b) + 2(\cos^2 b - \sin^2 b)} = \\
& \frac{2 \left(\frac{e^{2a} - e^{-2a}}{2} + 2i \sin b \cos b \right)}{2 \left(\frac{e^{2a} + e^{-2a}}{2} + \cos^2 b - \sin^2 b \right)} = \\
& \frac{\sinh 2a}{\cosh 2a + \cos 2b} + \frac{\sin 2b}{\cosh 2a + \cos 2b} i
\end{aligned}$$

Parameters

in	z	The ComplexNumber of which to compute the hyperbolic tangent
----	---	--

Returns

A [ComplexNumber](#) that is the hyperbolic tangent of a [ComplexNumber](#)

Definition at line 452 of file ComplexMath.cpp.

5.1.5 Member Data Documentation**5.1.5.1 const ComplexNumber<double> ComplexNumber::i [static]**

A public constant representing the value i .

This constant value is the imaginary unit, i , defined as $i^2 = -1$.

Definition at line 495 of file ComplexNumber.h.

5.1.5.2 T ComplexNumber::imaginary [private]

The value of the non-real part of the complex number.

This value is the non-real part of the complex number in rectangular form

Definition at line 517 of file ComplexNumber.h.

5.1.5.3 T ComplexNumber::real [private]

The value of the real part of the complex number.

This value is the real part of the complex number in rectangular form

Definition at line 509 of file ComplexNumber.h.

The documentation for this class was generated from the following files:

- [include/ComplexNumber.h](#)
- [include/ComplexMath.h](#)
- [src/ComplexNumber.cpp](#)

5.2 DivisionByZeroException Class Reference

A division by zero exception.

```
#include <MathExceptions.h>
```

Private Member Functions

- `const char * what () const throw ()`

5.2.1 Detailed Description

A division by zero exception.

"include/MathExceptions.h" This class represents a division by zero exception

5.2.2 Member Function Documentation

5.2.2.1 `const char* DivisionByZeroException::what () const throw ()` [`inline`, `private`]

Definition at line 75 of file MathExceptions.h.

The documentation for this class was generated from the following file:

- [include/MathExceptions.h](#)

5.3 LogarithmOfZeroException Class Reference

A logarithm of zero exception.

```
#include <MathExceptions.h>
```

Private Member Functions

- const char * [what](#) () const throw ()

5.3.1 Detailed Description

A logarithm of zero exception.

"include/MathExceptions.h" This class represents a logarithm of zero exception

5.3.2 Member Function Documentation

5.3.2.1 const char* [LogarithmOfZeroException::what](#) () const throw () [`inline`, `private`]

Definition at line 88 of file MathExceptions.h.

The documentation for this class was generated from the following file:

- include/[MathExceptions.h](#)

5.4 ZeroToComplexPowerException Class Reference

A zero to a complex power exception.

```
#include <MathExceptions.h>
```

Private Member Functions

- const char * [what](#) () const throw ()

5.4.1 Detailed Description

A zero to a complex power exception.

"include/MathExceptions.h" This class represents a zero to a complex power exception

5.4.2 Member Function Documentation

5.4.2.1 `const char* ZeroToComplexPowerException::what () const throw () [inline, private]`

Definition at line 101 of file MathExceptions.h.

The documentation for this class was generated from the following file:

- [include/MathExceptions.h](#)

5.5 ZeroToTheZerothPowerException Class Reference

A zero to the zeroth power exception.

```
#include <MathExceptions.h>
```

Private Member Functions

- `const char * what () const throw ()`

5.5.1 Detailed Description

A zero to the zeroth power exception.

"include/MathExceptions.h" This class represents a zero to the zeroth power exception

5.5.2 Member Function Documentation

5.5.2.1 `const char* ZeroToTheZerothPowerException::what () const throw () [inline, private]`

Definition at line 114 of file MathExceptions.h.

The documentation for this class was generated from the following file:

- [include/MathExceptions.h](#)

Chapter 6

File Documentation

6.1 include/ComplexMath.h File Reference

Elementary complex mathematical functions.

```
#include "ComplexNumber.h" #include "Namespace.h" #include <vector>
```

Namespaces

- namespace [math](#)
Math namespace.
- namespace [complex](#)
Complex namespace.

Defines

- #define [M_EI](#) 2.7182818284590452353602874713526625L
long double e
- #define [M_LOG2EI](#) 1.4426950408889634073599246810018921L
long double $\log_2 e$
- #define [M_LOG10EI](#) 0.4342944819032518276511289189166051L
long double $\log_{10} e$
- #define [M_LN2I](#) 0.6931471805599453094172321214581766L
long double $\ln 2$
- #define [M_LN10I](#) 2.3025850929940456840179914546843642L
long double $\ln 10$

- #define [M_PII](#) 3.1415926535897932384626433832795029L
long double π
- #define [M_PI_2I](#) 1.5707963267948966192313216916397514L
long double $\frac{\pi}{2}$
- #define [M_PI_4I](#) 0.7853981633974483096156608458198757L
long double $\frac{\pi}{4}$
- #define [M_1_PII](#) 0.3183098861837906715377675267450287L
long double $\frac{1}{\pi}$
- #define [M_2_PII](#) 0.6366197723675813430755350534900574L
long double $\frac{2}{\pi}$
- #define [M_2_SQRTPII](#) 1.1283791670955125738961589031215452L
long double $\frac{1}{\sqrt{\pi}}$
- #define [M_SQRT2I](#) 1.4142135623730950488016887242096981L
long double $\sqrt{2}$
- #define [M_SQRT1_2I](#) 0.7071067811865475244008443621048490L
long double $\frac{1}{\sqrt{2}}$

6.1.1 Detailed Description

Elementary complex mathematical functions.

Author

Matthew Krupcale

Version

0.1.2

Date

2010

This file contains the definitions or function prototypes for elementary complex mathematical functions. It also contains several mathematical constants that are used internally

Definition in file [ComplexMath.h](#).

6.1.2 Define Documentation

6.1.2.1 #define M_1_PII 0.3183098861837906715377675267450287L

long double $\frac{1}{\pi}$

Constant in long-double format using 128-bit IEEE quad.

$\frac{1}{\pi}$

Definition at line 162 of file ComplexMath.h.

6.1.2.2 #define M_2_PII 0.6366197723675813430755350534900574L

long double $\frac{2}{\pi}$

Constant in long-double format using 128-bit IEEE quad.

$\frac{2}{\pi}$

Definition at line 174 of file ComplexMath.h.

6.1.2.3 #define M_2_SQRTPII 1.1283791670955125738961589031215452L

long double $\frac{1}{\sqrt{\pi}}$

Constant in long-double format using 128-bit IEEE quad.

$\frac{1}{\sqrt{\pi}}$

Definition at line 186 of file ComplexMath.h.

6.1.2.4 #define M_EI 2.7182818284590452353602874713526625L

long double e

Constant in long-double format using 128-bit IEEE quad.

Euler's number, e , which can be represented in several ways, one of which follows,

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$$

Definition at line 61 of file ComplexMath.h.

6.1.2.5 #define M_LN10I 2.3025850929940456840179914546843642L

long double $\ln 10$

Constant in long-double format using 128-bit IEEE quad.

Natural logarithm of 10

Definition at line 109 of file ComplexMath.h.

6.1.2.6 `#define M_LN2I 0.6931471805599453094172321214581766L`

long double $\ln 2$

Constant in long-double format using 128-bit IEEE quad.

Natural logarithm of 2

Definition at line 97 of file ComplexMath.h.

6.1.2.7 `#define M_LOG10EI 0.4342944819032518276511289189166051L`

long double $\log_{10} e$

Constant in long-double format using 128-bit IEEE quad.

Logarithm base 10 of e

Definition at line 85 of file ComplexMath.h.

6.1.2.8 `#define M_LOG2EI 1.4426950408889634073599246810018921L`

long double $\log_2 e$

Constant in long-double format using 128-bit IEEE quad.

Logarithm base 2 of e

Definition at line 73 of file ComplexMath.h.

6.1.2.9 `#define M_PI_2I 1.5707963267948966192313216916397514L`

long double $\frac{\pi}{2}$

Constant in long-double format using 128-bit IEEE quad.

$\frac{\pi}{2}$

Definition at line 138 of file ComplexMath.h.

6.1.2.10 `#define M_PI_4I 0.7853981633974483096156608458198757L`

long double $\frac{\pi}{4}$

Constant in long-double format using 128-bit IEEE quad.

$$\frac{\pi}{4}$$

Definition at line 150 of file ComplexMath.h.

6.1.2.11 `#define M_PII 3.1415926535897932384626433832795029L`

long double π

Constant in long-double format using 128-bit IEEE quad.

π , which is defined as the ratio of a circle's circumference to its diameter,

$$\pi = \frac{C}{d}$$

Definition at line 126 of file ComplexMath.h.

6.1.2.12 `#define M_SQRT1_2I 0.7071067811865475244008443621048490L`

long double $\frac{1}{\sqrt{2}}$

Constant in long-double format using 128-bit IEEE quad.

$$\frac{1}{\sqrt{2}}$$

Definition at line 210 of file ComplexMath.h.

6.1.2.13 `#define M_SQRT2I 1.4142135623730950488016887242096981L`

long double $\sqrt{2}$

Constant in long-double format using 128-bit IEEE quad.

$$\sqrt{2}$$

Definition at line 198 of file ComplexMath.h.

6.2 include/ComplexNumber.h File Reference

Representation of a complex number.

```
#include "Namespace.h" #include <iosfwd>
```

Classes

- class [ComplexNumber](#)

A class template which supports complex numbers.

Namespaces

- namespace [math](#)

Math namespace.

- namespace [complex](#)

Complex namespace.

6.2.1 Detailed Description

Representation of a complex number.

Author

Matthew Krupcale

Version

0.1.2

Date

2010

This file contains the class template declaration that represents a complex number, its function prototypes as well as several declarations for overloaded operators

Definition in file [ComplexNumber.h](#).

6.3 include/MathExceptions.h File Reference

Classes that represent mathematical exceptions.

```
#include "Namespace.h" #include <exception>
```

Classes

- class [DivisionByZeroException](#)
A division by zero exception.
- class [LogarithmOfZeroException](#)
A logarithm of zero exception.
- class [ZeroToComplexPowerException](#)
A zero to a complex power exception.
- class [ZeroToTheZerothPowerException](#)
A zero to the zeroth power exception.

Namespaces

- namespace [math](#)
Math namespace.
- namespace [complex](#)
Complex namespace.

6.3.1 Detailed Description

Classes that represent mathematical exceptions.

Author

Matthew Krupcale

Version

0.1.0

Date

2010

This file contains classes that represent mathematical exceptions, such as domain errors

Definition in file [MathExceptions.h](#).

6.4 include/Namespace.h File Reference

Defines namespace macros.

Defines

- #define [BEGIN_NAMESPACE\(X\)](#) namespace X {
Begin a namespace.
- #define [END_NAMESPACE\(X\)](#) }
End a namespace.

6.4.1 Detailed Description

Defines namespace macros.

Author

Matthew Krupcale

Version

0.1.0

Date

2010

This file defines macros that can be used to begin an end namespace
Definition in file [Namespace.h](#).

6.4.2 Define Documentation

6.4.2.1 #define [BEGIN_NAMESPACE\(X \)](#) namespace X {

Begin a namespace.

This macro defines a new namespace with the name *X*
Definition at line 52 of file Namespace.h.

6.4.2.2 #define [END_NAMESPACE\(X \)](#) }

End a namespace.

This macro defines the end of a namespace with the name *X*
Definition at line 62 of file Namespace.h.

6.5 src/ComplexMath.cpp File Reference

Elementary complex mathematical functions.

```
#include "ComplexNumber.h" #include "ComplexMath.h" #include  
"MathExceptions.h" #include "Namespace.h" #include <cmath> ×  
#include <vector>
```

Namespaces

- namespace [math](#)
Math namespace.
- namespace [complex](#)
Complex namespace.

Defines

- `#define` [COMPLEX_MATH_CPP](#)

Functions

- `template<class T >`
`T` [abs](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
`T` [abs2](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
`T` [arg](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
`T` [logabs](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
[ComplexNumber](#)< T > [conjugate](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
[ComplexNumber](#)< T > [exp](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
[ComplexNumber](#)< T > [inverse](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
[ComplexNumber](#)< T > [log](#) (const [ComplexNumber](#)< T > &z)
- `template<class T >`
[ComplexNumber](#)< T > [log](#) (const T &x)
- `template<class T >`
[ComplexNumber](#)< T > [log10](#) (const [ComplexNumber](#)< T > &z)

- `template<class T >`
`ComplexNumber< T > log10 (const T &x)`
- `template<class T >`
`ComplexNumber< T > logb (const ComplexNumber< T > &z, const Complex-`
`Number< T > &b)`
- `template<class T >`
`ComplexNumber< T > logb (const ComplexNumber< T > &z, const T &b)`
- `template<class T >`
`ComplexNumber< T > logb (const T &x, const ComplexNumber< T > &b)`
- `template<class T >`
`ComplexNumber< T > logb (const T &x, const T &b)`
- `template<class T >`
`ComplexNumber< T > pow (const ComplexNumber< T > &a, const Complex-`
`Number< T > &b)`
- `template<class T >`
`ComplexNumber< T > pow (const ComplexNumber< T > &a, const T &b)`
- `template<class T >`
`ComplexNumber< T > pow (const T &a, const ComplexNumber< T > &b)`
- `template<class T >`
`ComplexNumber< T > sqrt (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > sqrt (const T &x)`
- `template<class T >`
`ComplexNumber< T > cbrt (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > nthRoot (const ComplexNumber< T > &z, const int n)`
- `template<class T >`
`std::vector< ComplexNumber< T > > nthRoots (const ComplexNumber< T >`
`&z, const int n)`
- `template<class T >`
`void nthRoots (const ComplexNumber< T > &z, std::vector< ComplexNumber<`
`T > > &roots, const int n)`
- `template<class T >`
`ComplexNumber< T > sin (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > cos (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > tan (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > csc (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > sec (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > cot (const ComplexNumber< T > &z)`

- `template<class T >`
`ComplexNumber< T > asin (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > asin (const T &x)`
- `template<class T >`
`ComplexNumber< T > acos (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > acos (const T &x)`
- `template<class T >`
`ComplexNumber< T > atan (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > atan (const T &x)`
- `template<class T >`
`ComplexNumber< T > acsc (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > acsc (const T &x)`
- `template<class T >`
`ComplexNumber< T > asec (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > asec (const T &x)`
- `template<class T >`
`ComplexNumber< T > acot (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > acot (const T &x)`
- `template<class T >`
`ComplexNumber< T > sinh (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > cosh (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > tanh (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > csch (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > sech (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > coth (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > asinh (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > asinh (const T &x)`
- `template<class T >`
`ComplexNumber< T > acosh (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > acosh (const T &x)`

- `template<class T >`
`ComplexNumber< T > atanh (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > atanh (const T &x)`
- `template<class T >`
`ComplexNumber< T > acsch (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > acsch (const T &x)`
- `template<class T >`
`ComplexNumber< T > asech (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > asech (const T &x)`
- `template<class T >`
`ComplexNumber< T > acoth (const ComplexNumber< T > &z)`
- `template<class T >`
`ComplexNumber< T > acoth (const T &x)`

6.5.1 Detailed Description

Elementary complex mathematical functions.

Author

Matthew Krupcale

Version

0.1.2

Date

2010

This file contains the implementations for elementary complex mathematical functions
Definition in file [ComplexMath.cpp](#).

6.5.2 Define Documentation

6.5.2.1 #define COMPLEX_MATH_CPP

Definition at line 25 of file [ComplexMath.cpp](#).

6.5.3 Function Documentation

6.5.3.1 `template<class T> T abs (const ComplexNumber< T> & z)` [\[related\]](#)

Definition at line 73 of file ComplexMath.cpp.

6.5.3.2 `template<class T> T abs2 (const ComplexNumber< T> & z)` [\[related\]](#)

Definition at line 80 of file ComplexMath.cpp.

6.5.3.3 `template<class T> ComplexNumber<T> acos (const ComplexNumber< T> & z)` [\[related\]](#)

Definition at line 361 of file ComplexMath.cpp.

6.5.3.4 `template<class T> ComplexNumber<T> acos (const T & x)` [\[related\]](#)

Definition at line 371 of file ComplexMath.cpp.

6.5.3.5 `template<class T> ComplexNumber<T> acosh (const ComplexNumber< T> & z)` [\[related\]](#)

Definition at line 497 of file ComplexMath.cpp.

6.5.3.6 `template<class T> ComplexNumber<T> acosh (const T & x)` [\[related\]](#)

Definition at line 507 of file ComplexMath.cpp.

6.5.3.7 `template<class T> ComplexNumber<T> acot (const ComplexNumber< T> & z)` [\[related\]](#)

Definition at line 426 of file ComplexMath.cpp.

6.5.3.8 `template<class T> ComplexNumber<T> acot (const T & x)` [\[related\]](#)

Definition at line 432 of file ComplexMath.cpp.

6.5.3.9 `template<class T> ComplexNumber<T> acoth (const ComplexNumber< T > & z)` [related]

Definition at line 569 of file ComplexMath.cpp.

6.5.3.10 `template<class T> ComplexNumber<T> acoth (const T & x)` [related]

Definition at line 575 of file ComplexMath.cpp.

6.5.3.11 `template<class T> ComplexNumber<T> acsc (const ComplexNumber< T > & z)` [related]

Definition at line 402 of file ComplexMath.cpp.

6.5.3.12 `template<class T> ComplexNumber<T> acsc (const T & x)` [related]

Definition at line 408 of file ComplexMath.cpp.

6.5.3.13 `template<class T> ComplexNumber<T> acsch (const ComplexNumber< T > & z)` [related]

Definition at line 541 of file ComplexMath.cpp.

6.5.3.14 `template<class T> ComplexNumber<T> acsch (const T & x)` [related]

Definition at line 547 of file ComplexMath.cpp.

6.5.3.15 `template<class T> T arg (const ComplexNumber< T > & z)` [related]

Definition at line 87 of file ComplexMath.cpp.

6.5.3.16 `template<class T> ComplexNumber<T> asec (const ComplexNumber< T > & z)` [related]

Definition at line 414 of file ComplexMath.cpp.

6.5.3.17 `template<class T> ComplexNumber<T> asec (const T & x)` [related]

Definition at line 420 of file ComplexMath.cpp.

6.5.3.18 `template<class T> ComplexNumber<T> asech (const ComplexNumber< T > & z)` [related]

Definition at line 553 of file ComplexMath.cpp.

6.5.3.19 `template<class T> ComplexNumber<T> asech (const T & x)` [related]

Definition at line 559 of file ComplexMath.cpp.

6.5.3.20 `template<class T> ComplexNumber<T> asin (const ComplexNumber< T > & z)` [related]

Definition at line 336 of file ComplexMath.cpp.

6.5.3.21 `template<class T> ComplexNumber<T> asin (const T & x)` [related]

Definition at line 347 of file ComplexMath.cpp.

6.5.3.22 `template<class T> ComplexNumber<T> asinh (const ComplexNumber< T > & z)` [related]

Definition at line 481 of file ComplexMath.cpp.

6.5.3.23 `template<class T> ComplexNumber<T> asinh (const T & x)` [related]

Definition at line 491 of file ComplexMath.cpp.

6.5.3.24 `template<class T> ComplexNumber<T> atan (const ComplexNumber< T > & z)` [related]

Definition at line 385 of file ComplexMath.cpp.

6.5.3.25 `template<class T> ComplexNumber<T> atan (const T & x)` [related]

Definition at line 396 of file ComplexMath.cpp.

6.5.3.26 `template<class T> ComplexNumber<T> atanh (const ComplexNumber< T > & z)` [related]

Definition at line 521 of file ComplexMath.cpp.

6.5.3.27 `template<class T> ComplexNumber<T> atanh (const T & x)` [related]

Definition at line 531 of file ComplexMath.cpp.

6.5.3.28 `template<class T> ComplexNumber<T> cbrt (const ComplexNumber< T > & z)` [related]

Definition at line 242 of file ComplexMath.cpp.

6.5.3.29 `template<class T> ComplexNumber<T> conjugate (const ComplexNumber< T > & z)` [related]

Definition at line 105 of file ComplexMath.cpp.

6.5.3.30 `template<class T> ComplexNumber<T> cos (const ComplexNumber< T > & z)` [related]

Definition at line 300 of file ComplexMath.cpp.

6.5.3.31 `template<class T> ComplexNumber<T> cosh (const ComplexNumber< T > & z)` [related]

Definition at line 445 of file ComplexMath.cpp.

6.5.3.32 `template<class T> ComplexNumber<T> cot (const ComplexNumber< T > & z)` [related]

Definition at line 330 of file ComplexMath.cpp.

6.5.3.33 `template<class T> ComplexNumber<T> coth (const ComplexNumber< T > & z)` [related]

Definition at line 475 of file ComplexMath.cpp.

6.5.3.34 `template<class T> ComplexNumber<T> csc (const ComplexNumber< T > & z)` [related]

Definition at line 318 of file ComplexMath.cpp.

6.5.3.35 `template<class T> ComplexNumber<T> csch (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 463 of file ComplexMath.cpp.

6.5.3.36 `template<class T> ComplexNumber<T> exp (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 111 of file ComplexMath.cpp.

6.5.3.37 `template<class T> ComplexNumber<T> inverse (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 118 of file ComplexMath.cpp.

6.5.3.38 `template<class T> ComplexNumber<T> log (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 128 of file ComplexMath.cpp.

6.5.3.39 `template<class T> ComplexNumber<T> log (const T & x)` [\[related\]](#)

Definition at line 137 of file ComplexMath.cpp.

6.5.3.40 `template<class T> ComplexNumber<T> log10 (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 146 of file ComplexMath.cpp.

6.5.3.41 `template<class T> ComplexNumber<T> log10 (const T & x)` [\[related\]](#)

Definition at line 152 of file ComplexMath.cpp.

6.5.3.42 `template<class T> T logabs (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 96 of file ComplexMath.cpp.

6.5.3.43 `template<class T> ComplexNumber<T> logb (const ComplexNumber< T > & z, const ComplexNumber< T > & b)` [related]

Definition at line 162 of file ComplexMath.cpp.

6.5.3.44 `template<class T> ComplexNumber<T> logb (const ComplexNumber< T > & z, const T & b)` [related]

Definition at line 168 of file ComplexMath.cpp.

6.5.3.45 `template<class T> ComplexNumber<T> logb (const T & x, const ComplexNumber< T > & b)` [related]

Definition at line 174 of file ComplexMath.cpp.

6.5.3.46 `template<class T> ComplexNumber<T> logb (const T & x, const T & b)` [related]

Definition at line 180 of file ComplexMath.cpp.

6.5.3.47 `template<class T> ComplexNumber<T> nthRoot (const ComplexNumber< T > & z, const int n)` [related]

Definition at line 253 of file ComplexMath.cpp.

6.5.3.48 `template<class T> std::vector<ComplexNumber<T>> nthRoots (const ComplexNumber< T > & z, const int n)` [related]

Definition at line 267 of file ComplexMath.cpp.

6.5.3.49 `template<class T> void nthRoots (const ComplexNumber< T > & z, std::vector< ComplexNumber< T >> & roots, const int n)`

Definition at line 275 of file ComplexMath.cpp.

6.5.3.50 `template<class T> ComplexNumber<T> pow (const ComplexNumber< T > & a, const ComplexNumber< T > & b)` [related]

Definition at line 186 of file ComplexMath.cpp.

6.5.3.51 `template<class T> ComplexNumber<T> pow (const ComplexNumber< T > & a, const T & b)` [\[related\]](#)

Definition at line 199 of file ComplexMath.cpp.

6.5.3.52 `template<class T> ComplexNumber<T> pow (const T & a, const ComplexNumber< T > & b)` [\[related\]](#)

Definition at line 208 of file ComplexMath.cpp.

6.5.3.53 `template<class T> ComplexNumber<T> sec (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 324 of file ComplexMath.cpp.

6.5.3.54 `template<class T> ComplexNumber<T> sech (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 469 of file ComplexMath.cpp.

6.5.3.55 `template<class T> ComplexNumber<T> sin (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 293 of file ComplexMath.cpp.

6.5.3.56 `template<class T> ComplexNumber<T> sinh (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 438 of file ComplexMath.cpp.

6.5.3.57 `template<class T> ComplexNumber<T> sqrt (const ComplexNumber< T > & z)` [\[related\]](#)

Definition at line 221 of file ComplexMath.cpp.

6.5.3.58 `template<class T> ComplexNumber<T> sqrt (const T & x)` [\[related\]](#)

Definition at line 232 of file ComplexMath.cpp.

6.5.3.59 `template<class T> ComplexNumber<T> tan (const ComplexNumber< T > & z)` [related]

Definition at line 307 of file ComplexMath.cpp.

6.5.3.60 `template<class T> ComplexNumber<T> tanh (const ComplexNumber< T > & z)` [related]

Definition at line 452 of file ComplexMath.cpp.

6.6 src/ComplexNumber.cpp File Reference

Representation of a complex number.

```
#include "ComplexNumber.h"          #include "MathExceptions.-
h" #include "Namespace.h" #include <iostream> #include
<cmath>
```

Namespaces

- namespace [math](#)
Math namespace.
- namespace [complex](#)
Complex namespace.

Defines

- #define [COMPLEX_NUMBER_CPP](#)

Functions

- `template<class T>`
[ComplexNumber< T > operator+](#) (const [ComplexNumber< T >](#) &lhs, const [ComplexNumber< T >](#) &rhs)
- `template<class T>`
[ComplexNumber< T > operator+](#) (const [ComplexNumber< T >](#) &lhs, const T &rhs)
- `template<class T>`
[ComplexNumber< T > operator+](#) (const T &lhs, const [ComplexNumber< T >](#) &rhs)

- `template<class T >`
`ComplexNumber< T > operator-` (const `ComplexNumber< T >` &lhs, const `ComplexNumber< T >` &rhs)
- `template<class T >`
`ComplexNumber< T > operator-` (const `ComplexNumber< T >` &lhs, const `T` &rhs)
- `template<class T >`
`ComplexNumber< T > operator-` (const `T` &lhs, const `ComplexNumber< T >` &rhs)
- `template<class T >`
`ComplexNumber< T > operator*` (const `ComplexNumber< T >` &lhs, const `ComplexNumber< T >` &rhs)
- `template<class T >`
`ComplexNumber< T > operator*` (const `ComplexNumber< T >` &lhs, const `T` &rhs)
- `template<class T >`
`ComplexNumber< T > operator*` (const `T` &lhs, const `ComplexNumber< T >` &rhs)
- `template<class T >`
`ComplexNumber< T > operator/` (const `ComplexNumber< T >` &lhs, const `ComplexNumber< T >` &rhs)
- `template<class T >`
`ComplexNumber< T > operator/` (const `ComplexNumber< T >` &lhs, const `T` &rhs)
- `template<class T >`
`ComplexNumber< T > operator/` (const `T` &lhs, const `ComplexNumber< T >` &rhs)
- `template<class T , class U >`
`bool operator==` (const `ComplexNumber< T >` &lhs, const `ComplexNumber< U >` &rhs)
- `template<class T , class U >`
`bool operator==` (const `ComplexNumber< T >` &lhs, const `U` &rhs)
- `template<class T , class U >`
`bool operator==` (const `T` &lhs, const `ComplexNumber< U >` &rhs)
- `template<class T , class U >`
`bool operator!=` (const `ComplexNumber< T >` &lhs, const `ComplexNumber< U >` &rhs)
- `template<class T , class U >`
`bool operator!=` (const `ComplexNumber< T >` &lhs, const `U` &rhs)
- `template<class T , class U >`
`bool operator!=` (const `T` &lhs, const `ComplexNumber< U >` &rhs)
- `template<class U >`
`std::ostream & operator<<` (`std::ostream &out`, const `ComplexNumber< U >` &z)

- `template<class U >`
`std::istream & operator>> (std::istream &in, ComplexNumber< U > &z)`

6.6.1 Detailed Description

Representation of a complex number.

Author

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Version

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Date

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This file contains the implementation of the class template that represents a complex number, its function template implementations as well as its overloaded operators

Definition in file [ComplexNumber.cpp](#).

6.6.2 Define Documentation

6.6.2.1 `#define COMPLEX_NUMBER_CPP`

Definition at line 25 of file [ComplexNumber.cpp](#).

6.6.3 Function Documentation

6.6.3.1 `template<class T, class U > bool operator!=(const ComplexNumber< T > & lhs, const ComplexNumber< U > & rhs)` [[related](#)]

Definition at line 355 of file [ComplexNumber.cpp](#).

6.6.3.2 `template<class T, class U > bool operator!=(const ComplexNumber< T > & lhs, const U & rhs)` [[related](#)]

Definition at line 361 of file [ComplexNumber.cpp](#).

6.6.3.3 `template<class T , class U > bool operator!=(const T & lhs, const ComplexNumber< U > & rhs)` [related]

Definition at line 366 of file ComplexNumber.cpp.

6.6.3.4 `template<class T > ComplexNumber<T> operator*(const ComplexNumber< T > & lhs, const ComplexNumber< T > & rhs)` [related]

Definition at line 280 of file ComplexNumber.cpp.

6.6.3.5 `template<class T > ComplexNumber<T> operator*(const ComplexNumber< T > & lhs, const T & rhs)` [related]

Definition at line 289 of file ComplexNumber.cpp.

6.6.3.6 `template<class T > ComplexNumber<T> operator*(const T & lhs, const ComplexNumber< T > & rhs)` [related]

Definition at line 294 of file ComplexNumber.cpp.

6.6.3.7 `template<class T > ComplexNumber<T> operator+(const ComplexNumber< T > & lhs, const ComplexNumber< T > & rhs)` [related]

Definition at line 246 of file ComplexNumber.cpp.

6.6.3.8 `template<class T > ComplexNumber<T> operator+(const ComplexNumber< T > & lhs, const T & rhs)` [related]

Definition at line 253 of file ComplexNumber.cpp.

6.6.3.9 `template<class T > ComplexNumber<T> operator+(const T & lhs, const ComplexNumber< T > & rhs)` [related]

Definition at line 258 of file ComplexNumber.cpp.

6.6.3.10 `template<class T > ComplexNumber<T> operator-(const ComplexNumber< T > & lhs, const ComplexNumber< T > & rhs)` [related]

Definition at line 263 of file ComplexNumber.cpp.

6.6.3.11 `template<class T> ComplexNumber<T> operator- (const ComplexNumber<T> & lhs, const T & rhs)` [\[related\]](#)

Definition at line 270 of file `ComplexNumber.cpp`.

6.6.3.12 `template<class T> ComplexNumber<T> operator- (const T & lhs, const ComplexNumber<T> & rhs)` [\[related\]](#)

Definition at line 275 of file `ComplexNumber.cpp`.

6.6.3.13 `template<class T> ComplexNumber<T> operator/ (const ComplexNumber<T> & lhs, const ComplexNumber<T> & rhs)` [\[related\]](#)

Definition at line 299 of file `ComplexNumber.cpp`.

6.6.3.14 `template<class T> ComplexNumber<T> operator/ (const ComplexNumber<T> & lhs, const T & rhs)` [\[related\]](#)

Definition at line 316 of file `ComplexNumber.cpp`.

6.6.3.15 `template<class T> ComplexNumber<T> operator/ (const T & lhs, const ComplexNumber<T> & rhs)` [\[related\]](#)

Definition at line 325 of file `ComplexNumber.cpp`.

6.6.3.16 `template<class U> std::ostream& operator<< (std::ostream & out, const ComplexNumber<U> & z)`

Sends a representation of this [ComplexNumber](#) into the ostream

Parameters

<code>in</code>	<code>out</code>	The ostream reference into which the ComplexNumber will be inserted
<code>in</code>	<code>z</code>	The ComplexNumber from which the data will be obtained

Returns

A reference to the modified ostream

Definition at line 371 of file `ComplexNumber.cpp`.

6.6.3.17 `template<class T, class U > bool operator==(const ComplexNumber< T > & lhs, const ComplexNumber< U > & rhs)` [related]

Definition at line 339 of file ComplexNumber.cpp.

6.6.3.18 `template<class T, class U > bool operator==(const ComplexNumber< T > & lhs, const U & rhs)` [related]

Definition at line 345 of file ComplexNumber.cpp.

6.6.3.19 `template<class T, class U > bool operator==(const T & lhs, const ComplexNumber< U > & rhs)` [related]

Definition at line 350 of file ComplexNumber.cpp.

6.6.3.20 `template<class U > std::istream& operator>> (std::istream & in, ComplexNumber< U > & z)`

Initializes a [ComplexNumber](#) from values extracted from the istream

Parameters

<code>in</code>	<code>in</code>	The istream reference from which the ComplexNumber will be extracted
<code>in</code>	<code>z</code>	The ComplexNumber into which the data will be inserted

Returns

A reference to the modified istream

Definition at line 383 of file ComplexNumber.cpp.