# Measurement of the Compton wavelength of the electron

Matthew Krupcale, Michael Anderson
*Department of Physics, Case Western Reserve University, Cleveland Ohio, 44106-7079*
23 February 2013

**Abstract**

A Compton spectrometer with two NaI scintillation detectors was used to measure the recoil electron energy as a function of gamma ray scattering angle for a beam of $662$ keV gamma rays. A reduced chi-squared value of $\chi_\nu^2 = 0.16$ indicates that the measured energies agree with the theoretical energy distribution. The mass of the electron and its Compton wavelength were determined from the energy distribution to be $m_e c^2 = 0.51 \pm 0.02$ MeV and $\lambda_C = (2.4 \pm 0.1) \times 10^{-12}$ m, respectively, which are in agreement with the accepted values of $0.510998928(11)$ MeV and $2.4263102389(16) \times 10^{-12}$ m [1,2].

**Introduction**

Compton scattering is an inelastic scattering of a photon by an electron, in which the original photon energy is divided between the recoil electron and the scattered photon. From conservation of momentum and energy, the shift in wavelength of the scattered photon is related to the Compton wavelength according to [3]

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta), \quad (1)$$

where $\lambda'$ and $\lambda$ are the scattered and original photon wavelengths, respectively, $h$ is Planck's constant, $m_e$ is the mass of the electron, $c$ is the speed of light and $\theta$ is the photon scattering angle relative to its incident direction. The quantity $\lambda_C = \frac{h}{m_e c}$ is the Compton wavelength of the electron. Since the energy of the photon is given by $E = h\nu = \frac{hc}{\lambda}$, Eq. 1 can be written in terms of the photon energies [3]

$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1 - \cos\theta)}. \quad (2)$$

Then the kinetic energy of the recoil electron is

$$E_{e^-} = E - E' = E\left(\frac{\frac{E}{m_e c^2}(1-\cos\theta)}{1 + \frac{E}{m_e c^2}(1-\cos\theta)}\right). \quad (3)$$

Thus, the mass of the electron can be determined by measuring the energy of the recoil electron as a function of photon scattering angle for a monoenergetic source of gamma rays; once the mass of the electron is known, its Compton wavelength is determined.

**Apparatus and Methods**

The recoil electron energy was measured by placing two separated NaI scintillation detectors in coincidence. Since Compton scattering is the predominant interaction mechanism for gamma rays of energies between several hundred keV and 5 MeV [4], gamma rays of energy

$E = 662$ keV from a $^{137}$Cs source were directed at the first detector. Some of the scattered gamma rays travel to the second detector at an angle $\Theta$ relative to the incident gamma ray direction. Since the two detectors are separated by only a few tens of centimeters, the two detector pulses are very nearly in coincidence. By selecting only the pulses from the first detector that are in coincidence with the second detector, only the single Compton scattering events for which the scattering angle is $\Theta$ are collected [4]. This reduces the energy spectrum from the first detector to a single peak within the Compton continuum whose energy is that of the recoil electron and is determined by the gamma ray scattering angle. Figure 1 demonstrates this arrangement.

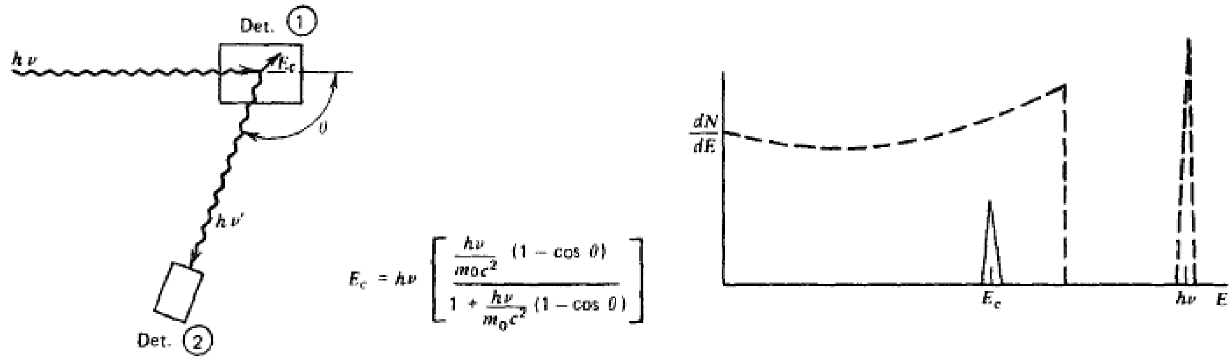


**Figure 1** Reproduced from reference [4]. The detector geometry is shown on the left; on the right, the typical spectrum for detector 1 is shown by the dashed curve, and the single peak of the recoil electron's energy resulting from coincident pulses from detector 2 is shown by the solid curve, where the Compton continiuum normally is.

Both NaI detectors were powered by high-voltage supplies at 900 V; their signals were amplified and discriminated against pulses below a minimum amplitude. The discriminator output logic pulses were used to determine coincidence, and a gate and delay generator connected to the coincidence unit regulated the input from detector 1 into the multichannel analyzer (MCA). The MCA was calibrated using the $^{22}$Na electron-positron annihilation peak at 511 keV and the $^{241}$Am 59.5 keV peak; the calibration was verified against the $^{137}$Cs 662 keV photopeak.

Uncertainties in the measured energies were minimized by maximizing the number of counts collected and maximizing the distance between the detectors, reducing the variation in scattering angles due to the finite sizes of the detectors. Energy spectra of detector 1 were measured with detector 2 placed at scattering angles between 30° and 135° in 15° increments for periods ranging from 4 hours to a couple of days, which was sufficient to acquire several thousand counts in the energy region of interest. The center-to-center distance between the detectors ranged from 51 to 66 cm, depending on the size limitations of a given scattering angle, and the distance from the source to the first detector was fixed at $L_1 = 30$ cm $\pm$ 1 cm.

**Results and Discussion**

The recoil electron peaks were fit with a Gaussian function above the background levels to determine the energy center, full width at half maximum (FWHM) and net number of counts, from which the statistical error in the energy is determined (see Appendix). Since there were several thousands of counts collected for each peak, systematic errors are by far the dominant contributor to the energy uncertainty. The systematic error arises from the calibration of the MCA as well as the selection of the region of interest for a particular peak measurement. By measuring the peak location using several different regions of interest for the same peak, we found that the number of counts within the region did not vary significantly, but the center of the peak often varied by $\delta_{E_{e^-},\mathrm{sys}} = 0.01$ MeV. Uncertainties in the scattering angle were estimated by taking the range of angles over which the scattered photon could be detected for a given geometry (see Appendix). Table 1 summarizes the measurement results.

**Table 1** Recoil electron energy measurements as a function of photon scattering angle. The distance between the detectors and the number of counts in each peak are also shown.

| Scattering angle, $\theta$ (degrees) | Center-to-center detector distance, $L_2$ (cm) | Recoil electron energy, $E_{e^-}$ (MeV) | Counts, $N$ |
|---|---|---|---|
| $30 \pm 4$ | $66 \pm 3$ | $0.10 \pm 0.01$ | $17600 \pm 500$ |
| $45 \pm 4$ | $66 \pm 3$ | $0.19 \pm 0.01$ | $2400 \pm 200$ |
| $60 \pm 4$ | $61 \pm 3$ | $0.26 \pm 0.01$ | $97000 \pm 2000$ |
| $75 \pm 4$ | $53 \pm 3$ | $0.32 \pm 0.01$ | $73000 \pm 2000$ |
| $90 \pm 4$ | $51 \pm 3$ | $0.37 \pm 0.01$ | $1400 \pm 200$ |
| $105 \pm 4$ | $53 \pm 3$ | $0.41 \pm 0.01$ | $35000 \pm 1000$ |
| $120 \pm 3$ | $61 \pm 3$ | $0.44 \pm 0.01$ | $20600 \pm 900$ |
| $135 \pm 4$ | $61 \pm 3$ | $0.46 \pm 0.01$ | $90000 \pm 1000$ |

Comparing the measured energies with the theoretical energy distribution of Eq. 3, there is a reduced chi-squared value of $\chi_v^2 = 0.16$, corresponding to a p-value of 0.996, indicating a strong agreement between the data and the expected distribution [5]. Figure 2 shows the measured data in comparison to the theoretical distribution.

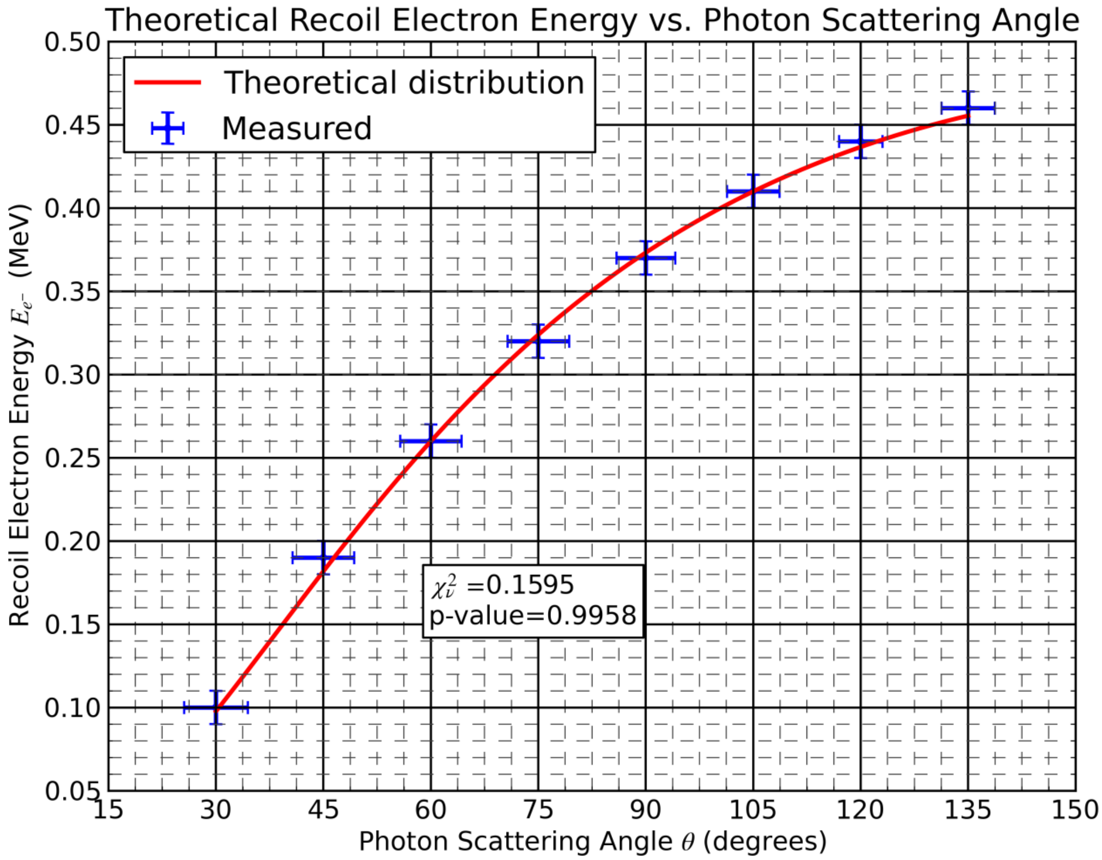


**Figure 2** Plot of the measured recoil electron energies (points shown in blue) in comparison to the theoretical recoil electron energy as a function of angle (shown by the red line).

The mass of the electron was found using orthogonal distance regression (ODR) by fitting the recoil electron energy distribution (Eq. 3) to the measured data, taking into account uncertainties in both the scattering angle and the energy. Figure 3 shows the resulting curve fit in comparison to the measured data. The best-fit parameter for the mass of the electron is $m_e c^2 = 0.51 \pm 0.02$ MeV, which is in agreement with the theoretical value of $0.510998928(11)$ MeV [1]. A reduced chi-squared value of $\chi^2_\nu = 0.056$ and p-value of $0.9998$ indicate that the data agrees with the fitted model.

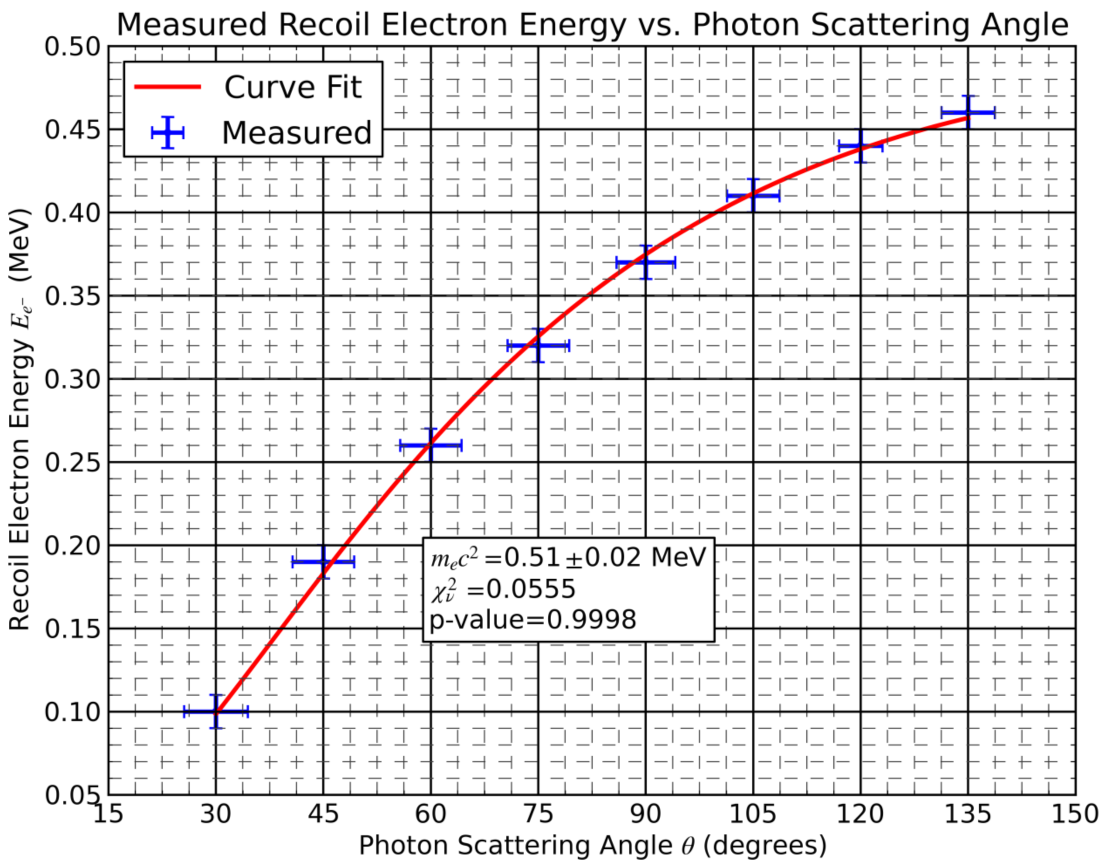


**Figure 3** Plot of the best-fit curve (shown in red) for the measured electron recoil energies (points shown in blue) as a function of photon scattering angle.

Then the Compton wavelength of the electron is

$$\lambda_C = \frac{h}{m_e c} = \frac{hc}{m_e c^2} = \frac{(4.135667516\times10^{-15}\ \text{eV}\cdot\text{s})(299792458\ \text{m}\cdot\text{s}^{-1})}{0.51\ \text{MeV}} = (2.4 \pm 0.1) \times 10^{-12}\ \text{m}$$

which is in agreement with the reference value of $2.4263102389(16) \times 10^{-12}$ m [2].

**References**


[1] *CODATA Fundamental Physics Constants*, http://physics.nist.gov/cgi-bin/cuu/Value?mec2mev

[2] *CODATA Fundamental Physics Constants*, http://physics.nist.gov/cgi-bin/cuu/Value?ecomwl

[3] A. C. Melissinos, *Experiments in Modern Physics*, (Academic Press, New York, 1966), p. 253.

[4] G. F. Knoll, *Radiation Detection and Measurement*, 3rd ed. (John Wiley, New York, 2000), pp. 307-317, 324-325.

[5] J. R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (University Science Books, California, 1997), pp. 271-272.

**Appendix: Uncertainties**

A Gaussian function's standard deviation is related to its full width at half maximum (FWHM) according to

$$\mathrm{FWHM} = 2\sqrt{2\ln 2}\,\sigma \Rightarrow \sigma_{E_{e^-}} = \frac{\mathrm{FWHM}}{2\sqrt{2\ln 2}}$$

For $N$ counts, the uncertainty in the recoil electron energy peak due to random statistical error is given by

$$\delta_{E_{e^-},\mathrm{rand}} = \frac{\sigma_{E_{e^-}}}{\sqrt{N}}$$

Then adding the systematic component of the uncertainty in quadrature, the total uncertainty in the recoil electron energy is given by

$$\delta_{E_{e^-}} = \sqrt{\delta^2_{E_{e^-},\mathrm{rand}} + \delta^2_{E_{e^-},\mathrm{sys}}} = \sqrt{\frac{\sigma^2_{E_{e^-}}}{N} + \delta^2_{E_{e^-},\mathrm{sys}}}$$

The uncertainty in the scattering angle was determined using a conservative estimate of the range of possible scattering angles within the detectors for a given geometry. Maximizing the distances between the source and detectors will minimize the uncertainty. Incident gamma rays from the source interact in the first detector at point $\mathbf{D}_1$ and scatter to produce a pulse in the second detector a displacement $\mathbf{D}_2$ relative to the Compton scattering location in the first detector. The angle between these two vectors is the scattering angle, as can be seen in the following figure.

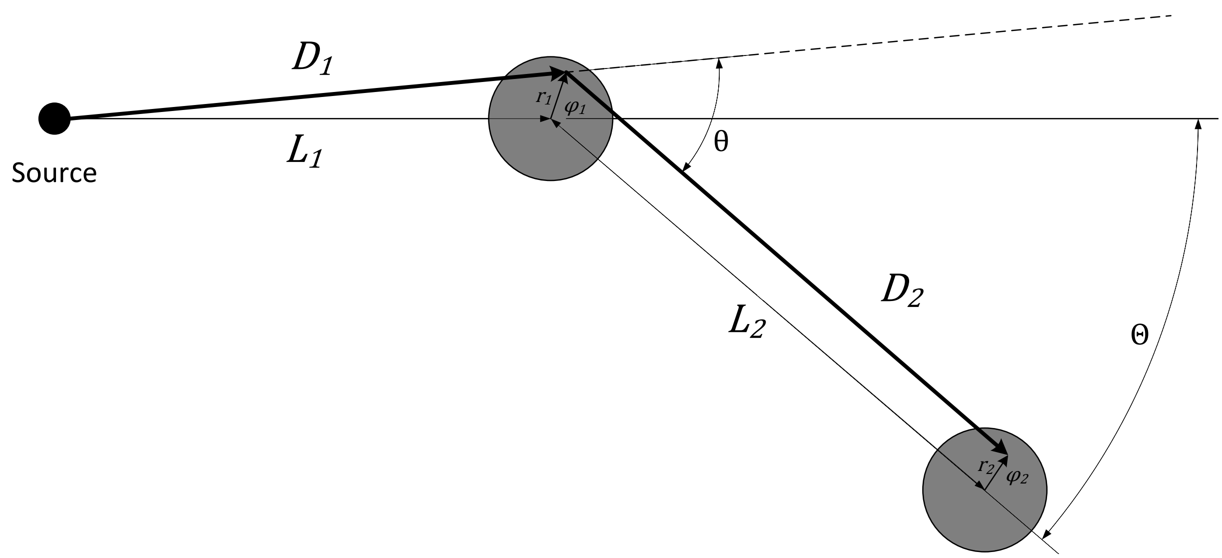


**Figure 4** Detector geometries used to calculate the uncertainty in the scattering angle.

The center of the first detector relative to the source is given by $\mathbf{L}_1 = L_1\hat{\mathbf{x}}$, and the vector from the center of the first detector to the center of the second detector is $\mathbf{L}_2 = L_2\cos\Theta\,\hat{\mathbf{x}} - L_2\sin\Theta\,\hat{\mathbf{y}}$. Defining the points within each detector in polar coordinates, we have $\mathbf{r}_i = r_i\cos\phi_i\,\hat{\mathbf{x}} + r_i\sin\phi_i\,\hat{\mathbf{y}}$. Then

$$\mathbf{D}_1 = \mathbf{L}_1 + \mathbf{r}_1 = (L_1 + r_1 \cos\phi_1)\hat{\mathbf{x}} + r_1 \sin\phi_1\, \hat{\mathbf{y}}$$

$$\begin{aligned}\mathbf{D}_2 = \mathbf{L}_1 + \mathbf{L}_2 + \mathbf{r}_2 - \mathbf{D}_1 &= \mathbf{L}_2 + \mathbf{r}_2 - \mathbf{r}_1 \\ &= (L_2 \cos\Theta + r_2 \cos\phi_2 - r_1 \cos\phi_1)\hat{\mathbf{x}} + (r_2 \sin\phi_2 - r_1 \sin\phi_1 - L_2 \sin\Theta)\hat{\mathbf{y}}\end{aligned}$$

and the angle between the vectors is given by their inner product:

$$\mathbf{D}_1 \cdot \mathbf{D}_2 = \|\mathbf{D}_1\|\|\mathbf{D}_2\| \cos\theta$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\mathbf{D}_1 \cdot \mathbf{D}_2}{\|\mathbf{D}_1\|\|\mathbf{D}_2\|}\right)$$

$$= \cos^{-1}\left(\frac{L_1 L_2 \cos\Theta - r_1(L_1 \cos\phi_1 - L_2 \cos(\Theta + \phi_1) + r_1) + r_2(L_1 \cos\phi_2 + r_1 \cos(\phi_1 - \phi_2))}{\sqrt{L_1^2 + 2L_1 r_1 \cos\phi_1 + r_1^2}\sqrt{L_2^2 + r_1^2 + 2L_2 r_2 \cos(\Theta + \phi_2) + r_2^2 - 2r_1(L_2 \cos(\Theta + \phi_1) + r_2 \cos(\phi_1 - \phi_2))}}\right)$$

The uncertainty in the angle then was given by

$$\delta_\theta = \frac{\max\theta - \min\theta}{2}$$

where $\theta$ is optimized over the areas of each detector for fixed $L_i$ and center-to-center angle $\Theta$.