Measurement of the band gaps of silicon and germanium

Matthew Krupcale, Devin Miles
Department of Physics, Case Western Reserve University, Cleveland Ohio, 44106-7079
4 May 2013

Abstract

The band gaps of silicon and germanium were measured using the I-V characteristics of BJTs to determine the reverse saturation current as a function of temperature from $-15°\mathrm{C}$ to $22°\mathrm{C}$. The band gaps of silicon and germanium were determined to be $E_g(\mathrm{Si}) = 1.13 \pm 0.02$ eV and $E_g(\mathrm{Ge}) = 0.65 \pm 0.03$ eV, which are in agreement with the reference values of 1.12 eV [1, 2] and 0.66 eV [1, 3] at $T = 300$ K, having less than two percent relative error.

Introduction

The band gap of a material is an energy region for which no wavelike electron orbitals exist and is given by the energy difference between the conduction and valence bands [3]. The band gap energy is one of the most important parameters in semiconductor physics [1] and has a strong influence on the characteristics and performance of optoelectronic devices [2] as well as the intrinsic carrier concentration. A p-n junction is of great importance in modern electronics and provides a basis for understanding many semiconductor devices [1]. The p-n junction diode and bipolar junction transistor (BJT) are examples of such devices, and the current through their junctions is given by [1]

$$I = I_0\left(\exp\left(\frac{qV}{kT}\right) - 1\right), \tag{1}$$

where I_0 is the reverse saturation current, q is the elementary charge, V is the applied voltage, T is the temperature, and k is the Boltzmann constant. The reverse saturation current is approximately proportional to the intrinsic carrier concentration and depends on the band gap and temperature according to [1, 4]

$$I_0 = AT^{3/2}\exp\left(-\frac{E_g}{kT}\right), \tag{2}$$

where A is a proportionality constant and E_g is the band gap. Eq. 2 can be rewritten in linear form as

$$\ln\left(\frac{I_0}{T^{3/2}}\right) = b + aT^{-1}, \tag{3}$$

where $b = \ln A$ and $a = -E_g/k$. Then the slope of the graph of the scaled saturation current versus the inverse temperature will yield the band gap:

$$E_g = -ka. \tag{4}$$

Apparatus and Methods

The current was measured through the base-emitter junctions of germanium and silicon BJTs as a function of applied voltage using a Keithley 487 picoammeter. The BJTs rested on a copper block inside an evacuated chamber with temperature sensors and thermoelectric coolers to control the temperature from $-15^{\circ}\mathrm{C}$ to $22^{\circ}\mathrm{C}$ in steps of approximately $0.5^{\circ}\mathrm{C}$. Current measurements typically reached up to $1.5\,\mathrm{mA}$ at voltages of up to $0.27\,\mathrm{V}$ or $0.79\,\mathrm{V}$ for germanium or silicon, respectively. As the temperature increased, smaller voltages were necessary to reach a given current. I-V characteristic measurements were made in both the cooling and heating directions so as to minimize the systematic error due to temperature variation.

Results and Discussion

The reverse saturation current was determined as a function of temperature for silicon and germanium by fitting Eq. 1 to the I-V characteristics at each temperature using orthogonal distance regression (ODR). A linear fit of the scaled saturation current in Eq. 3 versus the inverse temperature was then performed using ODR, from which the band gap was determined by Eq. 4. Errors in the measured voltage and current were estimated as the accuracy of the Keithley 487 picoammeter, while the temperature uncertainty was estimated to be $\delta_T = 0.1^{\circ}\mathrm{C}$, the resolution of the temperature sensor display. Figures 1 and 2 show the scaled saturation current versus inverse temperature for silicon and germanium, respectively.

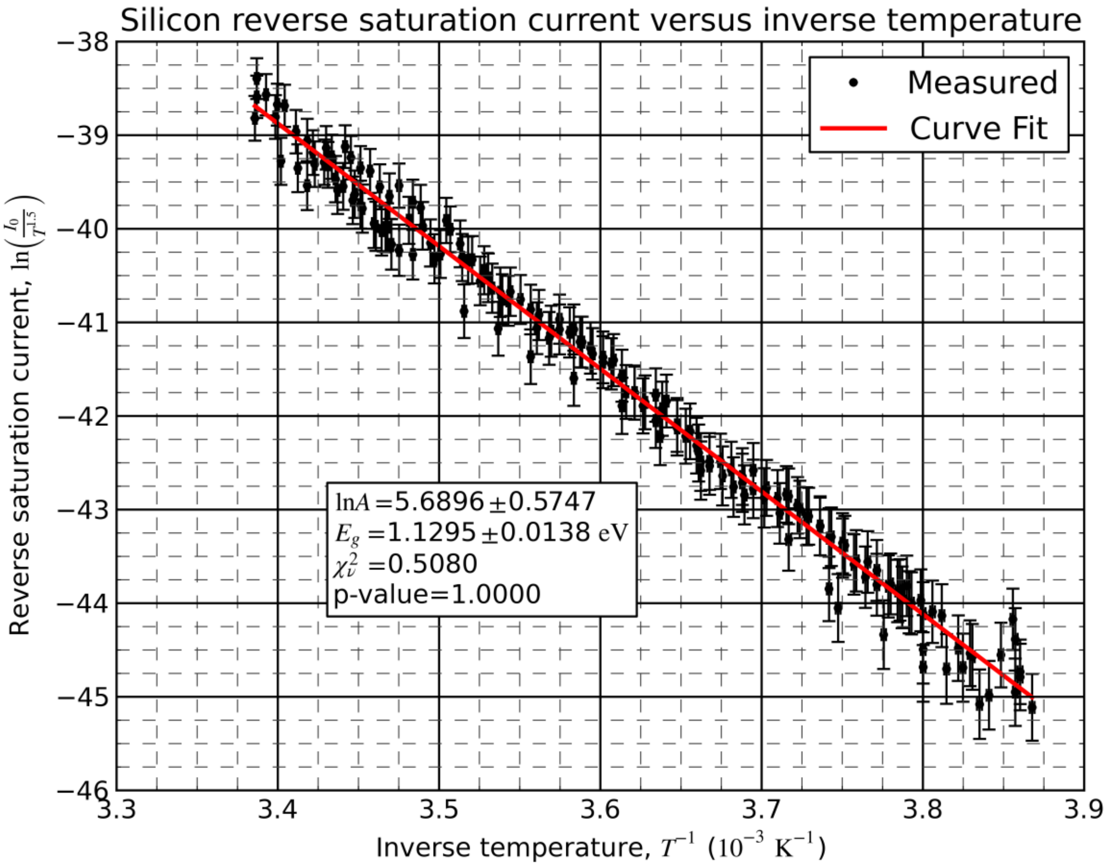

Figure 1 Logarithm of reverse saturation current versus inverse temperature for a silicon BJT. Measured values are shown in black, while the linear fit is shown in red. The reduced chi-squared and p-value indicate that the data agrees with the fitted model [5].

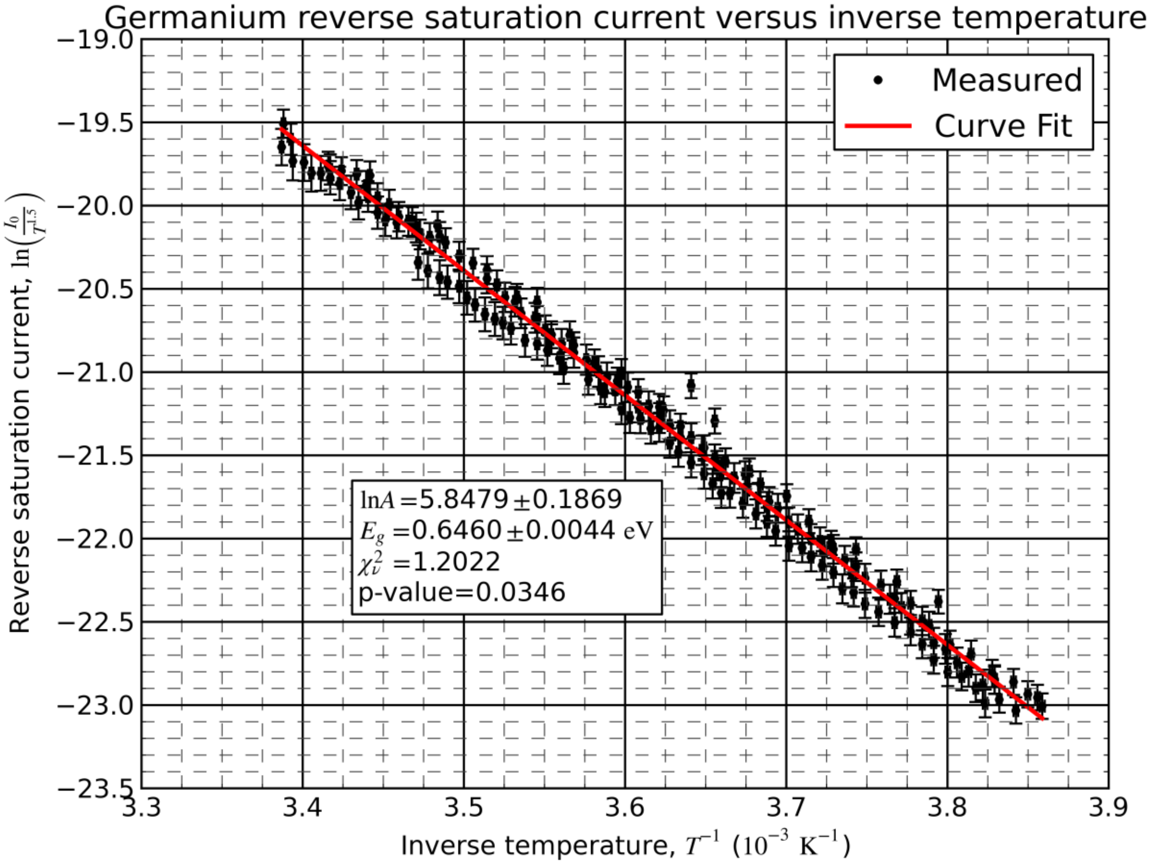

Figure 2 Logarithm of reverse saturation current versus inverse temperature for a germanium BJT. Measured values are shown in black, while the linear fit is shown in red. A reduced chi-squared ~1 indicates that the data agrees with the fitted model [5].

As shown in Figures 1 and 2, the measured band gaps of silicon and germanium from the linear fit are $E_g(\text{Si}) = 1.13 \pm 0.01\ \text{eV}$ and $E_g(\text{Ge}) = 0.646 \pm 0.004\ \text{eV}$. Thus, the measured band gap of silicon is within its uncertainty of the reference value of $1.12\ \text{eV}$ at $T = 300\ \text{K}$ [1, 2], while the measured band gap of germanium is not within its uncertainty of the reference value of $0.66\ \text{eV}$ at $T = 300\ \text{K}$ [1, 3]. However, these uncertainties do not account for any systematic error introduced by heating versus cooling the device, for instance. By performing curve fits on different sequences of data for cooling or heating from different days and taking the standard error in the slopes, we find systematic errors of $\delta_{E_g(\text{Si}),\text{sys}} = 0.02\ \text{eV}$ and $\delta_{E_g(\text{Ge}),\text{sys}} = 0.03\ \text{eV}$ for silicon and germanium, respectively. Adding these in quadrature with the random error, we have the band gaps $E_g(\text{Si}) = 1.13 \pm 0.02\ \text{eV}$ and $E_g(\text{Ge}) = 0.65 \pm 0.03\ \text{eV}$, which are now both within their uncertainties of the reference values.

In deriving Eq. 3, it was assumed that E_g was independent of temperature, but most semiconductor band gaps decrease with increasing temperature [1]. The variations of the theoretical band gaps over our temperature range are $\Delta E_g(\text{Si}) = -0.009\ \text{eV}$ and $\Delta E_g(\text{Ge}) =$

-0.014 eV, so the measured band gaps can be considered constant to a first approximation over our temperature range [6].

References

[1] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. (John Wiley & Sons, 1981), pp. 13, 15, 63, 87-88.

[2] C. C. Hu, *Modern Semiconductor Devices for Integrated Circuits*. (Prentice Hall, 2009), pp. 8-9.

[3] C. Kittel, *Introduction to Solid State Physics*, 8th ed. (John Wiley & Sons, 2005), pp. 163, 187.

[4] P. J. Collings, *Am. J. Phys*. 48 (3), 197-199 (1980).

[5] J. R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd ed. (University Science Books, California, 1997), pp. 271-272.

[6] J. W. Precker, M. A. da Silva, *Am. J. Phys*. 70 (11), 1150-1153 (2002).